General Analytical
Telescope Pointing Model

Written by

Marc W. Buie

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Introduction

Accurate pointing or positioning of a telescope is a common goal of any telescopic system that wishes to be productive. All motorized telescopes can be characterized by some coordinate system that is native to their axes of motion and are (re)positioned with motors. Unfortunately, it is arbitrarily difficult to make the mechanical system match any celestial coordinate system. Before the advent of computers, the precision of pointing and tracking was limited by the alignment of the mechanical system to the celestial sphere. Now that substantial computing power exists to control and operate telescopes, we no longer have to insist on mechanical perfection. Instead, using computer-based corrections mechanical imperfections can be modeled and thus removed. There are two basic styles of pointing correction models in vogue: (1) finite element grid corrections (with or without interpolation), and (2) analytical models. This document describes the implementation of an analytical model that is in use at all Lowell Observatory telescopes and at the ASU Braeside Observatory 16″ telescope. This model is a slight extension of the model originally developed at the University of Wyoming as published by Spillar *et al.*, *PASP*, **105**, pp 616-624 (1993). The following sections summarize the implementation of this model with my own extensions as it relates to the underlying mathematical structure of the software and to the process of determining the appropriate coefficients. For additional details on the model please read the Wyoming publication. Some of this information is transcribed (with corrections) from a memo written by Nat White (Lowell Observatory) on May 21, 1986.

Coordinate System

The fundamental coordinate system of the telescope is determined by its rotation axes. These axes are closely aligned with the Hour Angle and Declination axes. I will call these the raw or uncorrected positions. Of course, the Hour Angle is related to Right Ascension through the sidereal time but this step can be completely ignored for all the processing to be discussed. I will assume that you deal with RA only long enough to convert to hour angle and will not discuss this trivial transformation further.

I will represent the raw position of the telescope by (t, d) and the true position on the sky is (δ, τ) where t, τ are hour angle and d, δ are declination. Note that all celestial coordinates are assumed to be in the equinox of date.

Fitted Quantities

The analytical model attempts to provide pointing corrections based on physical misalignments and other mechanical distortions. These corrections are:

- $(\Delta d, \Delta t)$ zero-point offset
- (i) polar/declination axis non-orthogonality value
- (c) mis-alignment of optical and mechanical axes
- (e) tube flexure (droop) away from the zenith
- (γ) angular separation of true and instrumental pole
- (ϑ) angle between true meridian and line of true and instrumental poles
- (l) bending of declination axle (only for asymmetric mounts, ie., German Equatorial)

- (r) hour angle scale error (linear and quadratic terms)

The derivations of these equations can be found in the Wyoming paper.

Basic Equations

The difference between the raw and true positions is, by definition, the pointing error and is expressed by the following two equations. The units are arbitrary but I use arc-seconds internally in my software implementation of these transformations.

$$\delta - d = \Delta d - \gamma \cos(\tau - \vartheta) - e(\sin\varphi \cos\delta - \cos\varphi \sin\delta \cos\tau) \tag{1}$$

$$\begin{aligned} \tau - t = {} & \Delta t - \gamma \sin(\tau - \vartheta) \tan\delta + c \sec\delta - i \tan\delta \\ & + e \cos\varphi \sec\delta \sin\tau + l(\sin\varphi \tan\delta + \cos\delta \cos\tau) + r\tau \end{aligned} \tag{2}$$

The symbol, φ, stands for the latitude of the observatory. To apply these equations nine values must be detemined from observation: $i, c, e, \gamma, \vartheta, l, r, \Delta d,$ andΔt, typically from a simultaneous linear least-squares fit. To set this up for the fitting process, it is useful to transform the equations slightly. Let

$$\begin{aligned} a_0 &= \Delta d, \\ a_1 &= \gamma \cos\vartheta, \\ a_2 &= \gamma \sin\vartheta, \quad \text{and} \\ a_3 &= e \end{aligned} \tag{3}$$

which will be used to solve for the declination model. Similarly, we define the following for the hour angle model:

$$\begin{aligned} b_0 &= \Delta t, \\ b_1 &= c, \\ b_2 &= i, \\ b_3 &= l, \quad \text{and} \\ b_4 &= r. \end{aligned} \tag{4}$$

Note also the following convenient identities:

$$\begin{aligned} \gamma^2 &= a_1^2 + a_2^2, \\ \vartheta &= \cos^{-1}(\frac{a_1}{\gamma}), \\ \cos(\tau - \vartheta) &= \cos\tau \cos\vartheta + \sin\tau \sin\vartheta, \quad \text{and} \\ \sin(\tau - \vartheta) &= \sin\tau \cos\vartheta - \cos\tau \sin\vartheta. \end{aligned} \tag{5}$$

Finally we need the following additional definitions:

$$
\begin{aligned}
x &= \cos\tau, \\
y &= \sin\tau, \\
z &= \sin\varphi\cos\delta - \cos\varphi\sin\delta\cos\tau, \\
S &= \sec\delta, \\
T &= \tan\delta, \quad \text{and} \\
H &= (-a_1 y + a_2 x)T + a_3\cos\varphi S y
\end{aligned} \tag{6}
$$

which are to be calculated after solving for the constants in equation (1).

After substition, equations (1) and (2) now look like

$$\delta - d = a_0 - a_1 x - a_2 y - a_3 z \quad \text{and} \tag{7}$$

$$\tau - t = b_0 + b_1 S - b_2 T + H + b_3 q + b_4 \tau. \tag{8}$$

Equation (8) is shown in its symmetric representation. During the fitting process, the constant H is moved to the left side of the equation.

Collecting Data

To determine the pointing coefficients, you must make a series of measurements of the pointing error over the accessible sky. There are many ways to do this. The technique used for years at Lowell Observatory was to use an instrument with an eyepiece, then laboriously you would point the telescope near a bright star and center it in the eyepiece taking note of its catalog coordinates (converting to hour angle) and the raw telescope position. This process can take quite some time to build up enough useful positions on the sky and the accuracy of the observation is quite often determined by the observer's ability to center a star visually. A more modern technique in use now at the Lowell 31″ telescope is to take a series of CCD images on a regular grid on the sky. The instrumental coordinates are recorded with each frame and the true coordinates are determined from an astrometric fit to the images based on the USNO A2.0 star catalog. This process can generate very high quality data and the quality of the resulting model is limited by the mechanical quality and similarity to the physical model. The pointing model at the 31″ reaches 4 arc-second rms pointing quality over the entire sky. Regardless of how the data are obtained, the goal is to collect a large number of pointing error measurements over a large range of hour angle and declination. Ideally the range should cover the entire region accessible to the telescope so that the model will always bound the locations you wish to move the telescope to. In general, it takes at least 20-30 measurements all over the sky to get a good fit. More is clearly better if you have the patience for it.

IDL Software

Once the data are collected, it must be fit. I have written some IDL programs to facilitate the process. Unfortunately, due to the individual nature of all telescopes, these programs have not yet been generalized into a user-friendly fitting package. Successful operation of these programs and the generation of high-quality pointing maps depends on understanding this model and the terms used to fit that telescope. Not all terms are necessarily needed for all telescopes. But, you must determine with other means the need, or lack thereof, for any model term. If you don't need it, remove it and the solution will improve. Often an extra term will seem to be significant through cross-correlations with other terms. This is especially true of b_3 and b_4.

The following sub-sections attempt to explain the use of all of the pointing related programs within my IDL library. These programs provide some of the essential tools from which to derive a pointing map but they are not part of a highly polished system for handling the general problem. Instead, these should be taken as starting points and modified as needed for dealing with other facilities.

`rdpoint1.pro`

This program is used to read in the raw data file of observations to be fitted. This format happens to be generated directly by the Lowell Observatory MOVE computers. The file is in ascii tabular format with 6 columns. The columns are observed hour angle, observed declination, true hour angle, true declination, PPM number (not used but something must be present), and a flag (T/F whether the point should be included or not). In the input file, Hour angle is in decimal hours and Declination is in decimal degrees. The values returned are all converted to decimal degrees.

`pntcol.pro`

This program creates a file compatible with `rdpoint1` that is generated from CCD image headers and astrometric reductions of the images. The astrometry results are contained in a file, `centers.dat` which is created by `astrom.pro` when the images are measured astrometrically. Note that this data file will contain observed positions for each frame which is based on the pointing model in use at the time the data were collected. The previous model must first be removed from these data before the fitting process for the new model can begin.

`pntfix3.pro`

This program takes as input a given pointing model and raw positions and then applies the corrections and returns the true positions that would be computed based on that model. Since the pointing corrections are small, this can also be used to remove a pointing model by judicious algebra.

`pntfit2.pro`

This program takes the observed and true positions and performs a least-squares fit to determine the pointing model coefficients. All points are given equal weight and the observatory location is hard-coded to the position of Lowell Observatory. A number of plots are generated that show diagnostic information about the data and the goodness of fit. All of the variables described in equations (3) and (4) are fitted for and then converted to the constants need for equations (1) and (2). If you need to omit or add terms, the code must be modified accordingly.

`pntfit3.pro`

This is a modified version of pntfit2. The two differences are that the declination flexure term ($b_3 = l$) is disabled and a new term is added which depends on τ^2. The extra term in this model has only been needed for the Lowell 31″ telescope. Disabling the declination flexure should probably be done for all telescopes except for the Lowell Observatory Perkins Telescope (or other heavy german equatorial mounts).

Other Scripts

In addition to the library routines described above, there are other scripts used in this process. Since these scripts are much, much less general, they are not included in the library since they absolutely require editing and customization for other applications. These routines can be found with the standard IDL library routine directory on the Lowell Observatory ftp site. But, be forewarned, they will make no sense without some understanding of the material in this document. To get the sample code, download

`ftp://ftp.lowell.edu/pub/buie/idl/pointing/pointing.tgz`