Pergamon

Planet. Space Sci., Vol. 46, No. 11/12, pp. 1425–1432, 1998

0032–0633/98/$ - see front matter

PII: S0633(98)00023–3

The depletion of the Hecuba gap vs the long-lasting Hilda group

S. Ferraz-Mello,* T. A. Michtchenko, D. Nesvorný, F. Roig, and A. Simula

Universidade de São Paulo, Instituto Astronômico e Geofísico, Av. Miguel Stefano 4200, 04301-904-São Paulo, Brazil

Received 27 August 1997; revised 19 January 1998; accepted 19 January 1998

Abstract. This paper presents a comparative analysis of the 2/1 and 3/2 asteroidal resonances based on several analytical and numerical tools. The frequency map analysis was used to obtain a refined estimation of the chaotic transport. Fourier and wavelet analyses were used to construct the web of inner resonances and showed that they are the seat of the strongly unstable motion observed in the numerical simulations. The most regular regions in both resonances were classified. A fast symplectic mapping allowed a number of direct runs over 10^8 years of the orbits initially in these regions. The stability of orbits over the age of the solar system was discussed and compared to the distribution of the observed asteroids in both resonances. © 1998 Elsevier Science Ltd. All rights reserved

1. Introduction

The main-belt asteroid distribution is characterized by wide gaps at the 2/1 and 3/1 resonances with Jupiter (and narrower gaps at 5/2 and 7/3). On the other hand, an important group of relatively large asteroids is found at the 3/2 resonance. The problem of the gaps' formation is to be solved with the stringent condition that any mechanism able to deplete the gaps may not be efficient in removing asteroids from the 3/2 resonant group. This paper presents a comparative study of the results obtained by us for these two resonances. It does not aim to be a general review paper on these resonances; for that, the reader is referred to the recent review on the asteroidal resonances published by Michèle Moons (1997).

The gap in the 3/1 resonance could be explained in the frame of the planar restricted elliptic model Sun-Jupiter-asteroid (Wisdom, 1982; Ferraz-Mello & Klafke, 1991; see Ferraz-Mello *et al.*, 1996). The surfaces of section in this resonance showed three different regimes of motion and a strong chaos allowing an orbit to change from one regime to another. In these transitions, the eccentricity may reach high values, the asteroid becomes planet crossing, and a close approach to one of the inner planets expels it from the resonance. However, the same model failed to explain the 2/1 resonant gap. In this case, there are regular motions at medium eccentricities enclosing the chaotic region in low eccentricities and they do not allow the eccentricity to grow (Froeschlé and Scholl, 1976; Ferraz-Mello, 1994a). The regular motion in medium eccentricities is preserved even if long-period planetary perturbations of Jupiter's motion are added to the model (Morbidelli & Moons, 1993).

When a non-planar 4-body model of the 2/1 resonance is considered, signs of global chaoticity appear. The Lyapunov times are, generally, in the range $10^{3.5}$–$10^{5.5}$ years (Ferraz-Mello, 1994a) and the shortest time interval on which the global transitions of orbits happen is of the order of several million years (Wisdom, 1987). At variance, the Lyapunov times calculated in the regular region of the 3/2 resonance were by about 2 orders of magnitude larger (Ferraz-Mello 1994a,b) This was the first clue indicating that the main difference between the 2/1 and 3/2 resonances is quantitative: both resonances are globally chaotic, but unlike in the 2/1 resonance, the diffusion speed in the 3/2 resonance is small enough and did not allow the asteroids to be scattered from the resonance in the time elapsed since the formation of the solar system.

This was the starting point for the investigations reported in this article.

2. Resonant web and diffusion maps

The first task was to compute the position of the secondary, secular and Kozai resonances inside the 2/1 and 3/2 resonances in order to localize the sources of possible strong instabilities. There are several alternative ways of doing that. Figure 1 shows the comparison of the purely analytical method developed by Simula *et al.* (1998), based on the asymmetric expansion of the disturbing potential (Ferraz-Mello & Sato, 1989; Roig *et al.* 1998), and the semi-numerical method of Henrard (Henrard 1990; Nes-

Correspondence to: S. Ferraz-Mello. sylvio@usp.br

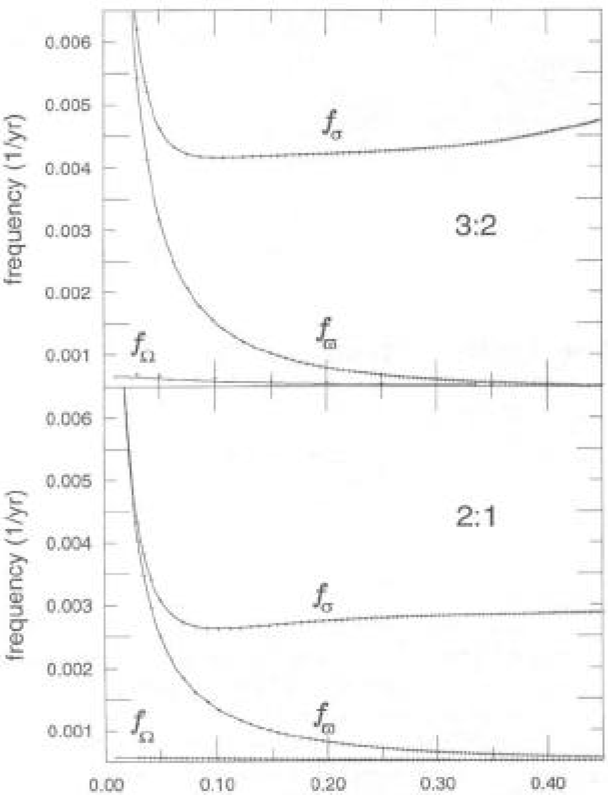

Fig. 1. Comparison of frequencies calculated on the pericentric branches for the 2/1 and 3/2 resonance with analytical and numerical methods (see text)

vorný, 1997). Using both methods, we computed three proper frequencies of the circular problem at the pericentric branch: (1) the libration frequency f_σ ($\sigma = (p+1)\lambda_{\rm Jup} - p\lambda - \varpi$; $p = 1$ for the 2/1 resonance and $p = 2$ for the 3/2 resonance); (2) the frequency of the perihelion longitude f_ϖ; and (3) the frequency of the node longitude f_Ω. The frequencies have been computed using a 3-D model, with Jupiter moving in a circular orbit. The correspondence of the results with the two methods is very good.

The calculated frequencies allowed us to find the location of the main inner resonances: $f_\sigma/f_\varpi = k$ (k integer) are the so-called secondary resonances, $f_\varpi = g_5$ is the secular resonance ν_5, $f_\Omega = s_6$ is the secular resonance ν_{16}, and $f_\varpi = f_\Omega$ is the Kozai resonance. This was done in the whole phase space (and not just on the pericentric branch) using a purely numerical method: the output of a numerical integration was submitted to a low-pass filter in order to cut out the short-period terms (less than 80–100 years) and the three proper frequencies were then computed by means of Fourier and wavelet transforms (Michtchenko and Ferraz-Mello, 1995; Michtchenko and Nesvorný, 1996). For the fixed initial value of the inclination ($I_0 = 3°$) and $\sigma_0 = \Delta\varpi_0 = \Delta\Omega_0 = 0$, these frequencies are smooth functions of two quantities: the initial eccentricity and semi-major axis.

Considering this evaluation of the proper frequencies, the webs of the main secular and secondary resonances were drawn in both the 2/1 and 3/2 resonances (Fig. 2). f_σ/f_ϖ denotes the secondary resonances, the thick lines indicate the borders of the mean-motion resonances, and the line denoted '*C*' indicates the pericentric branch of stationary solutions of the restricted problem.

The high-eccentricity region is dominated by the secular resonances: We show the location of the ν_5 secular resonance; the ν_6 and Kozai resonances are not plotted in the figure due to the fact that their numerical evaluation is complicated by strongly chaotic motion in these regions. Instead, we drawn the lower boundary of the overlapping ν_5, ν_6 and Kozai resonances (marked '*L*' in Fig. 2). All solutions with initial conditions chosen above this curve show strongly chaotic motion characterized by large increases of the eccentricity and inclination followed by fast escape of the asteroid from the resonance. This chaotic region is often called 'pericentric complex of secular resonances' (Morbidelli and Moons, 1993; Henrard *et al.* 1995).

In both 2/1 and 3/2 resonances, several important inner resonances occur below the lower boundary of the pericentric complex of secular resonances. The low-eccentricity regions are characterized by presence of the secondary resonances of the kind f_σ/f_ϖ (Lemaitre and Henrard, 1990). In the 2/1 resonance plane, we have plotted the $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$ and $\frac{5}{1}$ secondary resonances, whereas, in the 3/2 resonance, we have plotted the $\frac{2}{1}$, $\frac{3}{1}$ and $\frac{4}{1}$ secondary resonances. The dashed line shows the lower boundary of the region occupied by secondary resonances. This region is usually called 'complex of secondary resonances' and, as we will see later, it is dominated by strongly chaotic motion in the case of the 2/1 resonance.

Comparing the middle-eccentricity regions of the 2/1 and 3/2 resonances, the main difference between them is clearly noted. In the 2/1 resonance, this region is crossed by the very important ν_{16} secular resonance, while in the case of the 3/2 resonance, this secular resonance lies in the zone of the pericentric complex of secular resonances (due to this fact, the numerical determination of its position, in the 3/2 resonance, is difficult).

Now, having fulfilled the first task, the calculation of the resonant web, we turn our attention to the determination of the actual diffusion speed in both the 2/1 and 3/2 resonances using the frequency map analysis (Laskar, 1996). To achieve this, we define sets of initial conditions in the plane $I = 0$, $\sigma = 0$, $\Delta\varpi = 0$ and $\Delta\Omega = 0$. Each initial condition was integrated in the restricted four-body model with Jupiter and Saturn on their real orbits using the symmetric multistep method (Quinlan and Tremaine, 1990). The low-pass filter of Quinn *et al.* (1991) was applied to the variable $e \exp \imath\varpi$ ($\imath = \sqrt{-1}$), which was then decomposed into harmonics using the FMFT (Frequency Modified Fourier Transform) method of Šidlichovský and Nesvorný (1997) in two subsequent time intervals of 4×10^5 years with an overlap of 2×10^5 years.

The absolute value of the relative change of the leading frequency f_ϖ in the spectra (normalized to 10^5 years) was used as a measure of the local diffusion speed. The black squares in Fig. 3a (2/1 resonance) correspond to the trajectories with $\delta f_\varpi/f_\varpi > 10^{-2}$ and the small crosses to $10^{-2} > \delta f_\varpi/f_\varpi > 10^{-3}$ (the big gray squares forming the central gray area in high eccentricities and the gray areas at both sides of the rectangle, delimiting the high-eccentricity set, are highly chaotic orbits which quickly escaped

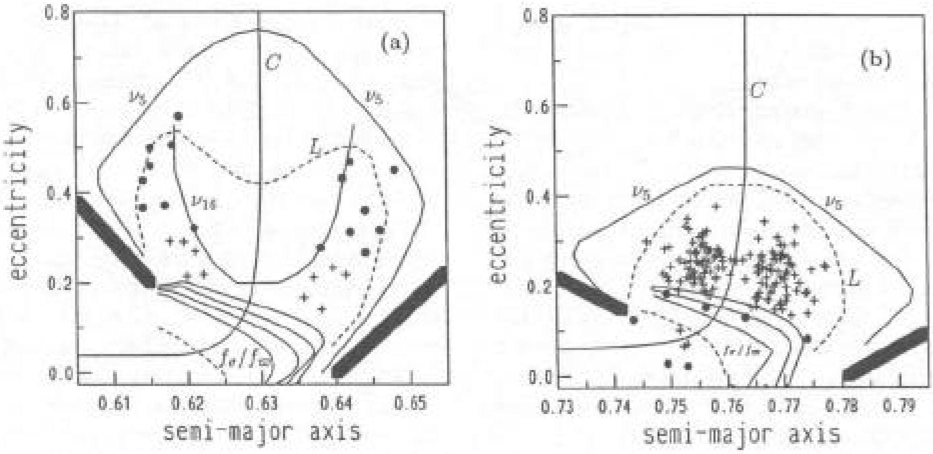

Fig. 2. (a) 2/1 resonance: (a, e)-plane of initial conditions for fixed $I_0 = 3^\circ$. The initial values of the angular elements are fixed at $\sigma_0 = \Delta\varpi_0 = \Delta\Omega_0 = 0$. The thick lines are borders of the resonant zone. The curve C is the pericentric branch of libration. L indicates the lower boundary of the secular resonances complex. The positions of the inner resonances (ν_5, ν_{16}, $f_\sigma/f_\varpi = \frac{2}{1}, \frac{3}{1}, \frac{4}{1}, \frac{5}{1}$) are indicated. Crosses and full circles depict the positions of the actual asteroids from Bowell's catalogue with regular and chaotic motion, respectively. (b) 3/2 resonance: same as (a), except that $f_\sigma/f_\varpi = \frac{2}{1}, \frac{3}{1}, \frac{4}{1}$ only

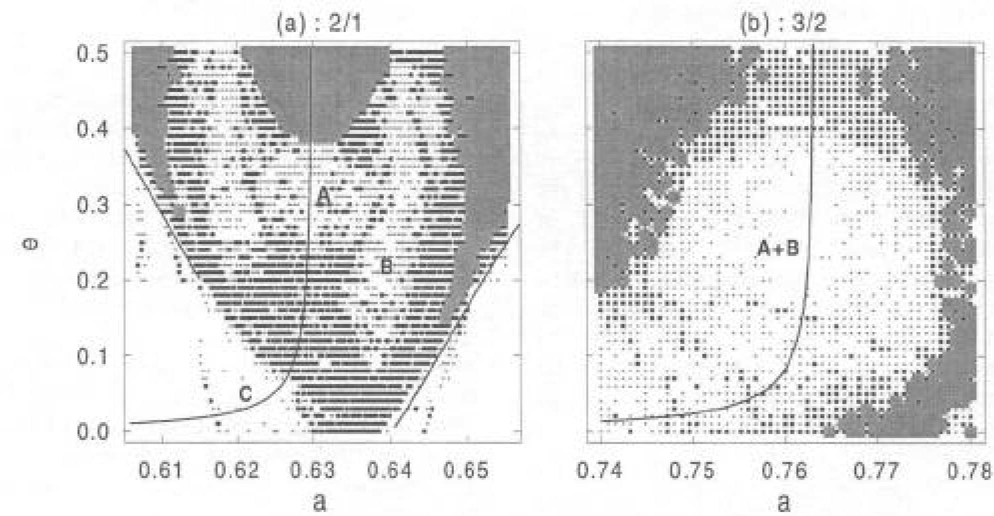

Fig. 3. The diffusion portrait of the 2/1 and 3/2 resonances in the representative plane with $I = 0$. For the 2/1 resonance, the black squares correspond to $\delta f_\varpi/f_\varpi > 10^{-2}$ in 10^5 years, small crosses to $10^{-2} > \delta f_\varpi/f_\varpi > 10^{-3}$ and voids to $\delta f_\varpi/f_\varpi < 10^{-3}$. For the 3/2 resonance, the black squares correspond to $\delta f_\varpi/f_\varpi > 10^{-2}$, small crosses to $10^{-2} > \delta f_\varpi/f_\varpi > 10^{-4}$ and voids to $\delta f_\varpi/f_\varpi < 10^{-4}$. The gray area corresponds to the escape orbits. (Taken from Nesvorný and Ferraz-Mello, 1997)

from the resonance). The rest, which was left blank, is the most stable area with $\delta f_\varpi/f_\varpi < 10^{-3}$. In the case of the 3/2 resonance, which is apparently more regular, the small crosses were reserved for $10^{-2} > \delta f_\varpi/f_\varpi > 10^{-4}$ (Fig. 3b).

The comparison of Figs 3a and 2a, centered at the 2/1 resonance, reveals several interesting characteristics of this resonance. The low-eccentricity chaos is associated with the overlap of the low-order secondary resonances. The chaos in high eccentricities ($e > 0.4$) originates from the ν_5, ν_6 and Kozai resonance and, together with the chaos near separatrices, enclose the chaotic, but comparatively more stable area. This more stable area is located at approximately $0.25 < e < 0.4$ in the center (A) and extends to a wider range in eccentricity when the amplitude of libration increases (B). The stable area B has a low-eccentricity V-shaped prolongation at roughly $a = 0.638$ going as low as $e = 0.1$. The typical relative frequency change (normalized to 10^5 years) ranges here from 10^{-2} to slightly less than 10^{-3}. The areas A and B are separated by a narrow ridge of faster diffusion, which can be identified with the secular resonance ν_{16} (Fig. 2a). This resonance forms an arc touching the complex of secondary resonances at about $e = 0.2$. Regions A and B are sometimes called 'central depleted zone' and 'Zhongguo family zone' (Michtchenko and Ferraz-Mello, 1997).

An additional detailed study showed that the most stable and compact regions in the resonance, where on average $\delta f_\varpi/f_\varpi < 10^{-3}$, are: 1) centered at $e = 0.3$ and $a = 0.63$, and 2) between 0.635 and 0.641 in semi-major axis and between 0.18 and 0.23 in eccentricity. These are the places where one should most likely expect to find asteroids (with low inclinations) lucky enough to survive for a long time inside the resonance.

In the case of the 3/2 resonance (Figs 3b and 2b), the most regular area A+B is not split into two parts by ν_{16} and it is much more regular than that of the 2/1 resonance. The whole area centered at $e = 0.25$ around the pericentric branch exhibits much slower diffusion than anywhere in the 2/1 resonance. The asteroids of the Hilda group are located in this central stable area of the 3/2 resonance. The diffusion over 10^5 years is $\delta f_{\varpi}/f_{\varpi} < 10^{-4}$ here, which would qualitatively mean less than a 10^{-2} change in 1 Gyr, is apparently not sufficient for substantial transitions in the phase space. Consequently, the observed asteroidal population should be dynamically primordial.

As in the case of the 2/1 resonance, we have computed the diffusion portraits in the model with four outer planets. This did not bring any substantial change to Fig. 3, and proved that the diffusion rates calculated in the model with only Jupiter and Saturn approximate well with reality.

The position of minor bodies obtained from Bowell's catalogue (Bowell *et al.* 1994) remaining inside the resonance boundaries over, at least, some thousand years, were recalculated on the corresponding (a, e)-planes. There are about 160 members of the Hilda group, 86 of which are numbered or multi-opposition ones, and about 100 asteroids in the 2/1 resonance, from which only 13 minor bodies are numbered or multi-opposition ones. In Fig. 2, their positions are marked either by crosses, if the asteroidal motion seems to be regular over 10 Myr, or by full circles, when asteroidal motion is chaotic. The consideration of the non-numbered real bodies must be done with caution due to their poorly determined orbits. Certainly these orbits need to be confirmed by means of additional observations; thus, in this paper, we consider only the numbered and multi-opposition asteroids of Bowell's catalogue.

3. Depleted regions

The distribution of the asteroids inside the resonant regions confirms our initial assumption about the importance of the inner resonances on the depletion process. In both resonances, the asteroids avoid the high-eccentricity region of strongly chaotic motion where the overlap of ν_5, ν_6 and Kozai resonances provokes large increase of eccentricity and eventual escape. It is worth mentioning that the chaotic motion in high eccentricities already appears in the restricted three-body model.

The low-eccentricity depleted region is caused by the overlap of the secondary resonances of the kind $f_{\sigma}/f_{\varpi} = k$ (secondary resonances complex); diffusion processes inside these regions are similar for both resonances. In Fig. 4, we show the chaotic diffusion of solutions with initial conditions inside the secondary resonances complexes. The rapid transition across the overlapping zones produces the slow chaotic diffusion of the orbits along the band of overlapping resonances. Starting near the pericentric branch and at small inclinations, the orbits suffer a random walk in amplitude of libration and in inclination. The escape of an asteroid occurs when its orbit approaches the resonance border indicated by the thick curve. Due to random character of this process, the escape time varies between several 10^7 and 10^8 years. For example, in Fig. 4, the escape happens after about 75 million years in the 2/1 resonance case, and, after only 10 million years in the 3/2 resonance case.

The extension of the frequency map analysis to a large range in inclination shows that the low- and high-eccentricity domains of fast diffusion, seen in Fig. 3, meet at high inclinations (Nesvorný & Ferraz-Mello, 1997). This phenomenon had already been detected by Henrard *et al.* (1995), who showed that this 'bridge' explains the results of the long-term integrations showing large increase of the eccentricity, after an important increase in inclination (Wisdom, 1987). The same behaviour was observed in many other simulations (Ferraz-Mello and Michtchenko, 1997). This 'bridge' is the second important diffusion path, which leads to emptying the secondary resonances complex.

As it is noted in Fig. 2, the important difference between the two mean-motion resonances is the presence of the ν_{16} secular resonance in the middle-eccentricity region of the 2/1 resonance, whereas, for 3/2 resonance, ν_{16} lies inside the high-eccentricity chaotic zone. The evolution of orbits near ν_{16} is characterized by a large increase of inclination (up to 20°) and chaotic diffusion along the ν_{16} resonant line; during this diffusion, the solution is driven to the domain of influence of the ν_5 and Kozai resonances, where a huge increase in eccentricity occurs.

In the case of the 3/2 resonance, the members of the Hilda group are concentrated in the central zone of the phase space that is free from the actions of the inner resonances. In the 2/1 resonance, the depleted central region, corresponding to solutions with small amplitude of libration, is actually a puzzle. In low-order analytical theories, this region seems to be stable (Henrard *et al.*, 1995). Numerical simulations done by Morbidelli (1996) point out that solutions with initial conditions inside this region can survive even 1 Gyr, notwithstanding the chaotic behavior.

4. Observed resonant objects

The analysis of about 30,000 orbits in Bowell's catalogue has shown 86 numbered and multi-opposition members in the Hilda group and 13 numbered and multi-opposition asteroids inside the Hecuba gap. The identification criterion was the oscillation of the asteroidal semi-major axis about the value corresponding to the exact resonance during, at least, 10^4 years.

The majority of the asteroids in Hilda group are pericentric librators and stay in the middle-eccentricity region ($0.1 < e < 0.35$) of the 3/2 resonance, far from the inner resonances. The dynamics of these asteroids is stable over very long time intervals. We have found two members of this group with clearly chaotic behaviour. In Fig. 2b, their positions are plotted by full circles. The diffusion due to the proximity to the secondary resonances complex (see Fig. 4b) may explain the non-existence of the low-eccentricity librating asteroids in the Hilda group.

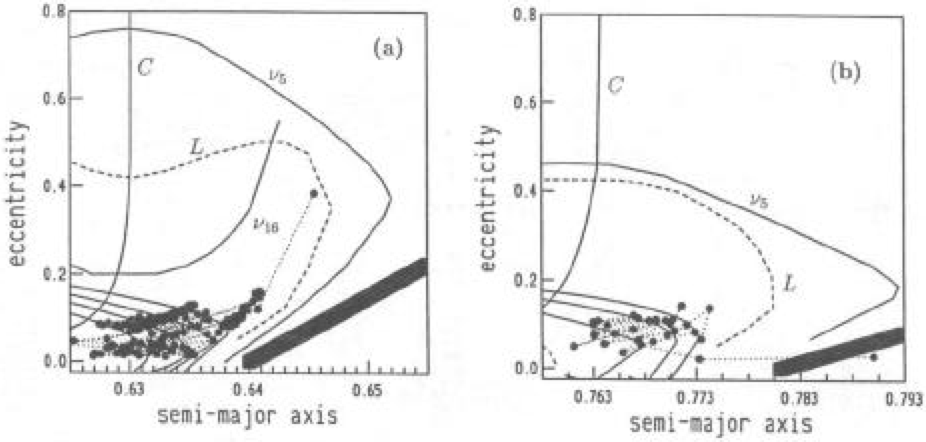

Fig. 4. (a) 2/1 resonance: Chaotic diffusion of one solution with initial conditions inside the secondary resonance zone. Escape occurs after 75 Myrs. (b) 3/2 resonance: same as (a), but escape occurs after 10 Myrs

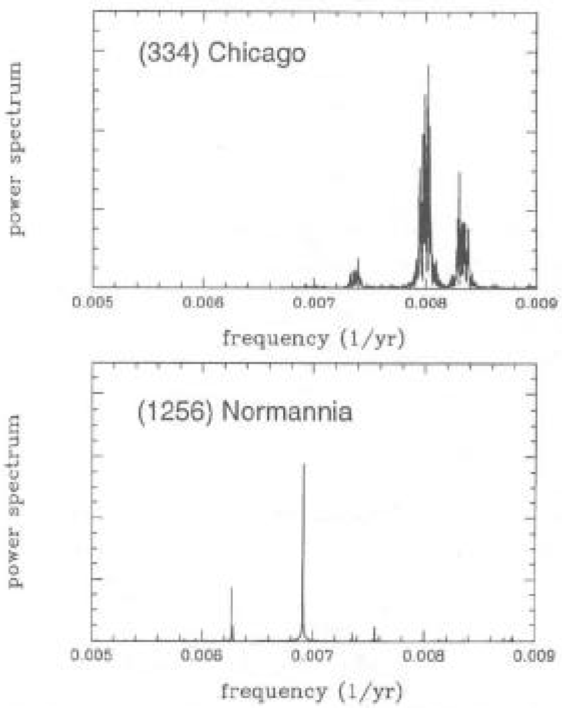

Fig. 5. Semi-major axis power spectra: (a) (334) Chicago; (b) (1256) Normannia

The six members of the Hilda group situated below the secondary resonances complex are alternators for which the angles σ and $\Delta\tilde{\omega}$ circulate and librate about 0, alternately. They are (334) Chicago, (1256) Normannia, (4196) Shuya, 1981 EF48, 1985 QX4 and 1996 AO3. Two asteroids of this group show chaotic behaviour ((334) Chicago and 1996 AO3). Figure 5 shows the power spectra of the semi-major axes of the (334) Chicago (a) and (1256) Normannia (b). The power spectrum of Chicago contains the broad-band components that are typical for chaotic motion; the power spectrum of (1256) Normannia is characterized by well-separated spectral lines.

In the 2/1 resonance there are seven numbered and six multi-opposition asteroids. (3789) Zhongguo, 1975 SX, 1990 TH7, 1993 SK3 and 1994 UD1 are members of the Zhongguo group. Two asteroids, (4177) 1987 SS1 and 1981 EX11, are in the ν_{16} secular resonance, (3688) Navajo is near ν_{16}; the other asteroids have clearly transition orbits.

5. Effect of the 5/2 great inequality

To determine the importance of different dynamical mechanisms acting in the 2/1 and 3/2 resonances, we need to perform simulations of a great number of initial conditions over a long interval of time (of the order of the age of the Solar System). In order to overcome the practical difficulties of this task, we constructed a symplectic mapping for the asteroidal resonant motion (Ferraz-Mello, 1997; Roig, 1997), using an implicit first-order scheme similar to that of Hadjidemetriou (1991). In this mapping, the averaged disturbing potential was replaced by the first terms of its high-eccentricity asymmetric expansion (Ferraz-Mello and Sato, 1989; Roig *et al.* 1998). This mapping allows the inclusion or exclusion of each of the long-period variations of the orbit of Jupiter arising from the main secular perturbations of the other planets and from the great inequality (GI) associated with the near 5/2 commensurability between the mean motions of Jupiter and Saturn. The main result of the performed experiments was to discover the important role played by the GI on the acceleration of the diffusive mechanisms, specially in the central region of the 2/1 resonance. Figure 6 shows the initial conditions of asteroids that escape from the 2/1 resonance after a large eccentricity increase ($e > 0.5$) in a 10^8-years simulation using a 3-D model. In Fig. 6a we took into account only the main secular variations in the orbit of Jupiter, and in Fig. 6b we added the main GI-associated perturbations in mean longitude, eccentricity and perihelion. The influence of these GI-associated perturbations was studied with more detail performing some 'ad-hoc' changes in the value of the GI-period. We cover an interval of GI-periods from 200–1000 yr (the actual value is 883 years) and we observed that, in the central region of the 2/1 resonance, the number of escapes increase when the GI-period comes closer to the libration period in the resonance which is ~440 yr (Fig.

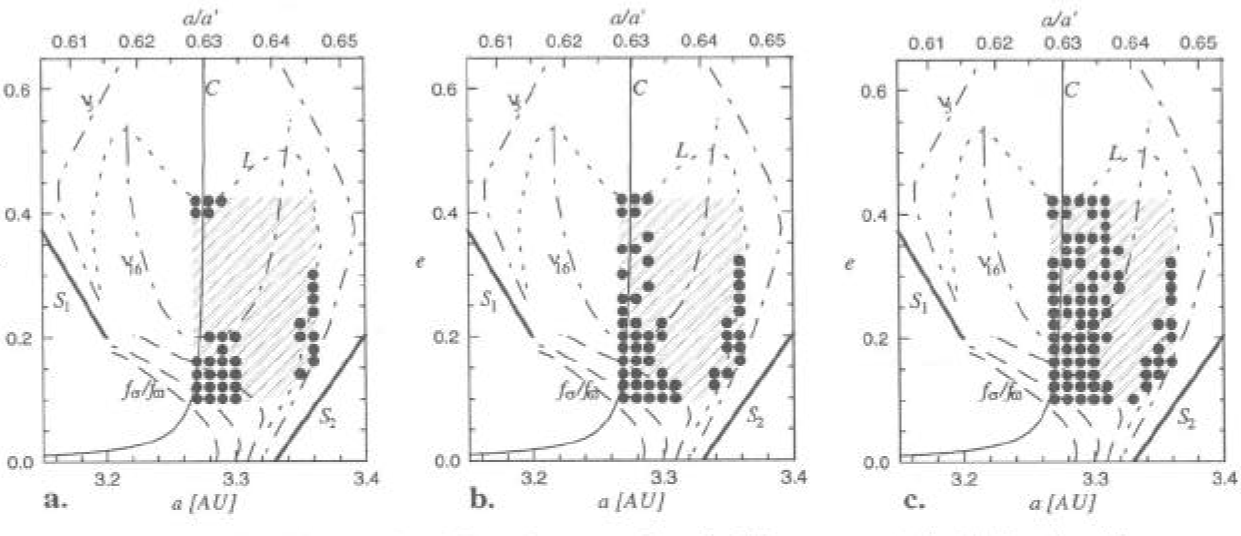

Fig. 6. Results of the mapping. Objects that escape from the 2/1 resonance (circles) in less than 10^8 years. The dashed region indicates the grid of initial conditions tested. We also show the pericentric branch (C), the separatrices (S_1, S_2), and the position of some secular and secondary resonances. The curve L is the lowest limit of the pericentric complex of secular resonances. (a) Three-dimensional model considering just secular variations in the orbit of Jupiter. (b) The same model, but now adding the variations arising from the GI. (c) The same as (b), but now adjusting the GI-period to the value of the libration period in the resonance (about 440 years)

6c). A similar mechanism seems to act in the 3/2 resonance; however, it is not efficient for the depletion of the Hilda group.

The influence of the GI-associated perturbations in the 2/1 resonance was also verified with precise numerical integrations using a four-body model (Michtchenko and Ferraz-Mello, 1997). In this case, the variations in the GI-period were done by means of slight changes in the initial orbit of Jupiter. In fact, it is enough to move Jupiter along its orbit by few tens of degrees, keeping all other osculating elements unaltered, to decrease the GI-period to the half of its present value. It is worth stressing that the shift affects only the critical GI-frequency ($2n_{\rm Jup}-5n_{\rm Sat}$). All other important frequencies have only very small variations, which do not have any effect in the final qualitative result (see Michtchenko and Ferraz-Mello, 1997). Choosing the value of the GI-period comparable to the value of the libration period allowed simulations to be made in which the diffusion was accelerated. In these simulations, objects escape in a few tens of Myr. Experiments were done with a GI-period of about 440 years, but it is worth noting that the Fourier spectrum of the asteroidal orbit modifies significantly even with smaller changes in the GI-period (for instance 650 years). Consequently, some mechanisms of planetary migration (Fernandez and Ip, 1996) may have affected the period of the GI and thus, contributed to a fast depletion of the Hecuba gap in the past. However, it must be emphasized that, in the central part of the 2/1 resonance, the diffusion exists even with the present value of the GI-period.

In Fig. 7, two solutions corresponding to both the current and the modified configuration of the Jupiter–Saturn system are shown. The initial conditions of both solutions differ only by the value of the initial mean anomaly of Saturn. The first solution in Fig. 7a, corresponding to the actual positions of Jupiter and Saturn, shows a very slow outward diffusion. In the other one (Fig. 7b), corresponding to the modified system, the diffusion is faster, and the orbit reaches the region of the ν_{16} secular resonance after 10 Myr; the asteroid escapes from the 2/1 resonance in about 25 Myr.

6. Conclusions

There is a slight difference in the diffusion timescales of the resonances 3/2 and 2/1. Even if the dynamics of the two resonances is very similar, the diffusion timescale in the 3/2 resonance is much smaller (by a factor larger than 10, or even 100). The mechanisms that have depleted the 2/1 resonance will take a time much longer than the age of the solar system to produce similar effects in the 3/2 resonance. The results obtained with the frequency map analysis thus confirmed the conclusions of Ferraz-Mello (1994a,b) concerning the robustness of the Hilda group of asteroids.

The Hecuba gap in the 2/1 resonance is not completely depleted. This gap is only partial. There are, at least, 5 asteroids on very regular orbits: (3789) Zhongguo, 1975 SX, 1990 TH7, 1993 SK3 and 1994 UD1. These asteroids are small (the largest of them, Zhongguo, is certainly less than 20 km in diameter); at variance, most of the known asteroids in the 3/2 resonance have more than 50 km (some of them reaching about 200 km). There are eight other asteroids in the 2/1 resonance in irregular orbits. They belong to the resonance only temporarily and are bound to escape in times in the range 10^4–10^7 years.

There is a well-established structure in the 2/1 resonance defined by the secondary and secular resonances. We may define several domains with different dynamics:

(A) The low-diffusion domain in the middle of the res-

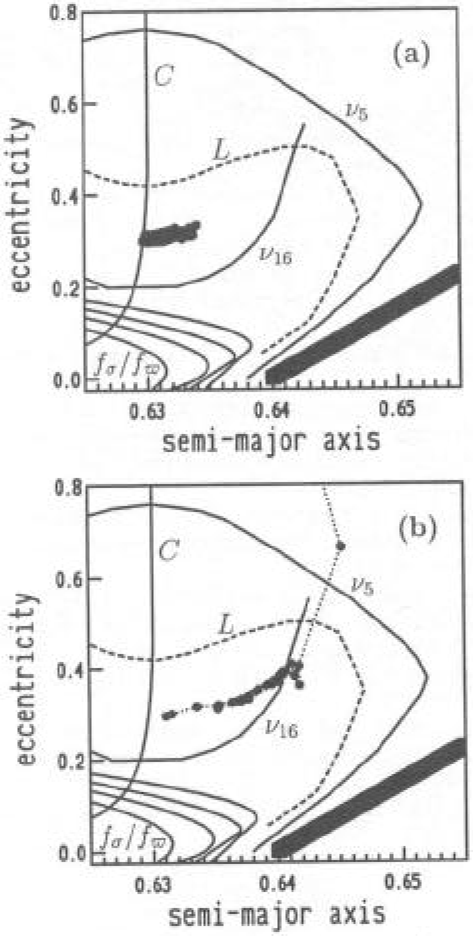

Fig. 7. Chaotic diffusion of two solutions starting near pericentric branch, for 2/1 resonance. (a) Solution calculated for actual position of Jupiter and Saturn over 200 Myrs. (b) Solution for which the value of the mean semi-major axis of Jupiter altered of 10^{-3} AU. Escape of asteroid occurs after 25 Myr. (Taken from Michtchenko and Ferraz-Mello, 1997)

onance. It is located above the line of the ν_{16} resonance and no asteroid was found inside it. This domain is strongly affected by GI-associated perturbations of Jupiter orbit.

(B) The low-diffusion domain below the ν_{16} line and between this line and the lower boundary of the pericentric complex of secular resonances. In the middle of the region, there are 5 asteroids of Zhongguo group.

(C) The region below the secondary resonances. This region is usually considered as not belonging to the 2/1 resonance since the critical angle is no longer librating there. However, there is no dynamical (or topological) separation between these orbits and the adjacent ones where the critical angle librates (Ferraz-Mello, 1984). This domain is much more regular than the regions (A) and (B), but few asteroids are there.

The existence of asteroids in (B) and their complete absence in (A) is an open question. Morbidelli (1996) conjectured that this fact may be related to the existence of the Themis family formed by fragmentation of a larger asteroid some 1 Gyr ago. Indeed, the sizes of the asteroids of Zhongguo group are estimated as generally no larger than 10–20 km.

The structure of the 3/2 resonance has an important difference with respect to the one described above: the secular resonance ν_{16} is above the Kozai resonance and does not split the low-diffusion region in two parts as in the 2/1 resonance. The asteroids of the 3/2 resonance are in the domain that corresponds to A + B.

Simulations with a modified GI-period have given a handful of new results: (a) The areas (A) and (B) in the 2/1 resonance crunch to about half-size, their parts in larger eccentricities being the most affected; (b) Similar simulations in the 3/2-resonance show no appreciable modifications (simulations with the symplectic map only showed some minor effects when the GI-period was lowered to below 300 years). The contrasting results in the 2/1 and 3/2 resonance must be due to the very different spectral composition of these resonances even in the case of Jupiter in a fixed ellipse (see Michtchenko and Ferraz-Mello, 1995); (c) Simulations over hundreds of million years show typical patterns for 2/1-solutions starting with very small libration amplitude (that is, in area (A)): During a long time, the orbital eccentricity and inclination have only small changes, while the amplitude of libration increases; when this amplitude becomes large and approaches the ν_{16} resonance line, the inclination begins to increase and the asteroid reaches the so-called pericentric complex formed by the Kozai and ν_5 resonances, where the variations of the elements become erratic and the asteroid leaves the resonance.

Acknowledgements. We acknowledge the support of CAPES, CNPq and FAPESP (Thematic Project 'Capture, Escape and Permanence in Resonances in the Solar System').

References

Bowell, E., Muinonen, K. and Wasserman, L. H. (1994) A public-domain asteroid orbit database. *IAU Symposium* **160,** ed. A. Milani *et al.*, pp. 477–481.

Fernández, J. A. and Ip, W. H. (1996) Orbital expansion and resonant trapping during the late accretion stages of the outer planets. *Planet. Sp. Science* **44,** 431–439.

Ferraz-Mello, S. and Klafke, J. C. (1991) A model for the study of very-high-eccentricity asteroidal motion. The 3:1 resonance, in *Predictability, Stability and Chaos in N-Body Dynamical Systems*, ed. A. E. Roy, Plenum Press, New York, pp. 177–184.

Ferraz-Mello, S. (1994a) Dynamics of the 2:1 asteroidal resonance. *Astron. J.* **108,** 2330–2337.

Ferraz-Mello, S. (1994b) Kirkwood Gaps and Resonant Groups. *IAU Symposium* **160,** ed. A. Milani *et al.* pp. 175–188.

Ferraz-Mello, S. (1997) A symplectic mapping approach to the study of the stochasticity in asteroidal resonances. *Cel. Mech. Dyn. Astr.* **65,** 421–437.

Ferraz-Mello, S. and Sato, M. (1989) The very-high-eccentricity asymmetric expansion of the disturbing function near resonances of any order. *Astron. Astrophyts.* **225,** 541–547.

Ferraz-Mello, S., Klafke, J. C., Michtchenko, T. A. and Nesvorný, D. (1996) Chaotic transitions in resonant asteroidal dynamics. *Cel. Mech. Dyn. Astr.* **64,** 93–105.

Ferraz-Mello, S. and Michtchenko, T. A. (1996) Orbital evolution of asteroids in the Hecuba gap, in *The dynamical behav-*

iour of our planetary system, ed. R. Dvorak *et al.* Kluwer, Dordrecht, pp. 377–384.

Froeschlé, C. and Scholl, H. (1976) On the dynamical topology of the Kirkwood gaps. *Astron. Astrophys.* **48,** 389–393.

Hadjidemetriou, J. (1991) Mapping models for Hamiltonian systems with application to resonant asteroidal motion, in *Predictability, Stability and Chaos in N-body Dynamical Systems*, ed. A. E. Roy, Plenum Press, New York, pp. 157–175.

Henrard, J. (1990) A semi-numerical perturbation method for separable hamiltonian systems. *Celest. Mech.* **49,** 43–68.

Henrard, J., Watanabe, N. and Moons, M. (1995) A bridge between secondary and secular resonances inside the Hecuba gap. *Icarus* **115,** 336–346.

Laskar, J. (1996) Introduction to frequency map analysis, in *Proceedings of the NATO Advances Study Institute 3DHAM95*, Kluwer, Dordrecht, in press.

Lemaitre, A. and Henrard, J. (1990) Origin of the chaotic behaviour in the 2/1 Kirkwood gap. *Icarus* **83,** 391–409.

Michtchenko, T. A. and Ferraz-Mello, S. (1995) Comparative study of the asteroidal motion in the 3/2 and 2/1 resonances with Jupiter. I. Planar model. *Astron. Astrophys.* **303,** 945–963.

Michtchenko, T. A. and Nesvorný, D. (1996) Wavelet analysis of the resonant asteroidal motion. *Astron. Astrophys.* **313,** 674–678.

Michtchenko, T. A. and Ferraz-Mello, S. (1997) Escape of asteroids from the Hecuba gap, *Planet. Sp. Science* **45,** 1587–1593.

Moons, M. (1997) Review of the dynamics in the Kirkwood gaps. *Cel. Mech. Dyn. Astr.* **65,** 175–204.

Morbidelli, A. and Moons, M. (1993) Secular resonances in mean-motion commensurabilities. The 2/1 and 3/2 cases. *Icarus* **102,** 316–332.

Morbidelli, A. (1996) On the Kirkwood gap at the 2/1 commensurability with Jupiter, *Astron. J.* **111,** 2453–2461.

Nesvorný, D. (1997) Ph.D. Thesis, University of São Paulo, Brazil.

Nesvorný, D. and Ferraz-Mello. S. (1997) On the asteroidal population of the first-order jovian resonances. *Icarus* **329,** 339–349.

Quinlan, G. D. and Tremaine, S. (1990) Symmetric multistep methods for the numerical integration of planetary orbits. *Astron. J.* **100,** 1694–1700.

Quinn, T. R., Tremaine, S. and Duncan, M. (1991) A three million year integration of the Earth's orbit. *Astron. J.* **101,** 2287–2305.

Roig, F. (1997) M.Sc. Dissertation, University of São Paulo, Brazil.

Roig, F., Simula, A., Ferraz-Mello, S. and Tsuchida, M. (1998) The high-eccentricity asymmetric expansion of the disturbing function for non-planar resonant problems. *Astron. Astrophys.* **130,** 247–258.

Šidlichovský, M. and Nesvorný, D. (1997) Frequency modified Fourier transform and its application to asteroids. *Cel. Mech. Dyn. Astr.* **65,** 137–148.

Simula, A., Ferraz-Mello, S. and Giordano, C. (1998) On high-eccentricity small-amplitude librations. in *The Dynamics of Small Bodies in the Solar System: A major key to Solar System Studies*. eds. A. E. Roy and B. Steves, Kluwer, Dordrecht, in press.

Wisdom, J. (1982) The origin of the Kirkwood gaps: A mapping for asteroidal motion near the 3/1 commensurability. *Astron. J.* **87,** 577–593.

Widsom, J. (1987) Chaotic dynamics in the Solar system. *Icarus* **72,** 241–275.