

On the V-type asteroids outside the Vesta family

I. Interplay of nonlinear secular resonances and the Yarkovsky effect: the cases of 956 Elisa and 809 Lundia

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Abstract. Among the largest objects in the main belt, asteroid 4 Vesta is unique in showing a basaltic crust. It is also the biggest member of the Vesta family, which is supposed to originate from a large cratering event about 1 Gyr ago (Marzari et al. 1996). Most of the members of the Vesta family for which a spectral classification is available show a V-type spectra. Due to their characteristic infrared spectrum, V-type asteroids are easily distinguished. Before the discovery of 1459 Magnya (Lazzaro et al. 2000) and of several V-type NEA (Xu 1995), all the known V-type asteroids were members of the Vesta family. Recently two V-type asteroids, 809 Lundia and 956 Elisa, (Florczak et al. 2002) have been discovered well outside the limits of the family, near the Flora family. We currently know 22 V-type asteroids outside the family, in the inner asteroid belt (see Table 2). In this work we investigate the possibility that these objects are former family members that migrated to their current positions via the interplay of Yarkovsky effect and nonlinear secular resonances.

The main dynamical feature of 956 Elisa and 809 Lundia is that they are currently inside the $2(g - g_6) + s - s_6$ (z_2 by Milani & Knežević 1993) secular resonance. Our investigations show that members of the Vesta dynamical family may drift in three-body and weak secular resonances until they are captured in the strong z_2 secular resonance. Only asteroids with diameters larger than 16 km can remain in one of the three-body or secular resonances long enough to reach the region of the z_2 resonance. This two-step mechanism of capture into the z_2 resonance could explain: i) the current resonant orbits of 956 Elisa and 809 Lundia; ii) why their size is significantly larger than that of the typical member of the Vesta family; and iii) provide a lower limit on the Vesta family age. We believe that other V-type asteroids could have followed the same path, and could currently be inside the z_2 resonance.

In an forthcoming article of this series we will investigate the role that other mechanisms of dynamical mobility, such as close encounters with massive asteroids, may have played in causing the current orbital distribution of the remaining 20 other V-type asteroids.

Key words. minor planets – asteroids – celestial mechanics

1. Introduction

Asteroid families are thought to be formed as the results of collisional events in the main belt. After the break-up or cratering event on the parent body, the fragments are ejected and form a cluster identifiable in proper element space. Unlike instantaneous orbital elements, that respond to short-period perturbations, proper elements remain nearly constant in time for conservative systems (Lemaitre 1993). However, when chaotic diffusion or non gravitational forces are considered, proper elements may change significantly. Since the family formation, some of the members may therefore have drifted in proper element space, and could no longer be recognizable as part of

the family. By dynamical family we define the family as identified on the basis of the current proper orbital elements. The problem of identifying possible past members of asteroid families has been the subject of many recent studies (Bottke et al. 2003). In this paper we will concentrate on the case of the Vesta family.

The Vesta family, one of the largest in the inner asteroid belt, is believed to have originated in a cratering event that excavated a basin on the surface of 4 Vesta. According to Marzari et al. (1996) this cratering event occurred ≈ 1 Gyr ago (the age is poorly constrained). Using the hierarchical clustering method (Zappalà et al. 1990), and a cutoff of 62 m/s, Mothé-Diniz et al. (2005) estimated that the Vesta family presently accounts for

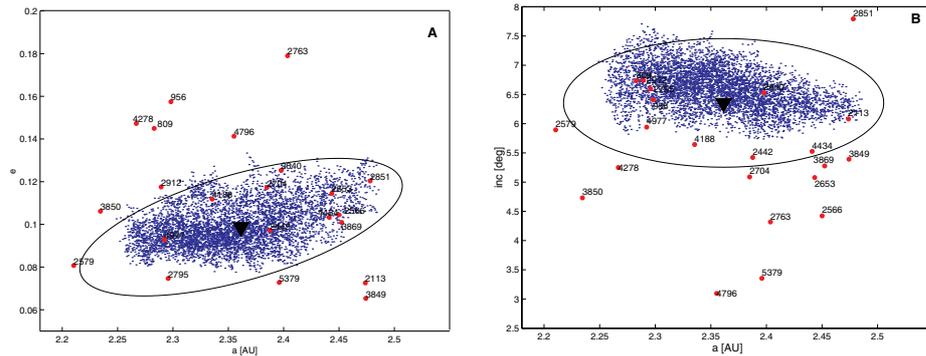


Fig. 1. Location in proper element space ($a - e$, **a**); $a - i$, **b**) of the 5112 asteroids (small dots) members of the Vesta family (Mothè-Diniz et al. 2005). The black triangle shows the location of 4 Vesta itself, while the ellipse displays the 600 m/s level of maximum ejection velocity. The other dots show the locations of the 22 V-type asteroids that are not members of the family.

about 5000 members, mostly with diameters of less than 4 km. The typical ejection velocities with respect to 4 Vesta necessary to reach the edges of this family are of the order of 600 m/s (see Fig. 1).

Most of the members of the Vesta dynamical family (including 4 Vesta itself) present a V-type spectrum, which is characterized by a moderately steep red slope shortwards of $0.7 \mu\text{m}$ and a deep absorption band longwards of $0.75 \mu\text{m}$. This kind of spectra is associated with a basaltic surface composition (McCord 1970; Bus 2002; Duffard et al. 2004). Until the discovery of 1459 Magnya (Lazzaro 2000) and of several V-type NEAs (Bus & Binzel 2002, and references within), the only known V-type asteroids were members of the Vesta family. Recently Florczak et al. (2002) discovered two inner belt asteroids located well outside the edges of the family (809 Lunda and 956 Elisa) that show a V-type spectrum. Today 22 V-type asteroids that are not thought to be members of the dynamical family are known in the inner belt (Fig. 1, Table 2). Most of these asteroids have orbits corresponding to ejection velocities with respect to 4 Vesta larger than 1 km s^{-1} , the maximum possible ejection velocity expected to be produced in the cratering event.

The V-type asteroids outside the family can either be former members that dynamically migrated to their current orbital positions after family formation, or fragments of primordial large V-type asteroids other than 4 Vesta. This second scenario opens interesting perspectives about the primordial main belt. However, before considering it, we should be certain that all possible migration mechanisms from the Vesta family have been dismissed as improbable or impossible.

Among the possible mechanisms of dynamical migration there is evolution in secular resonances. A secular resonance occurs when the frequency of variation of the longitude of pericenter ϖ (g), or longitude of node Ω (s), becomes nearly equal to an eigenfrequency (or a combination of eigenfrequencies) of the system of coupled planetary orbits. The main linear secular resonances (of order 2), such as $g - g_5$, $g - g_6$, and $s - s_6$, have been recognized in the asteroid belt for a long time (Williams & Faulkner 1981). But, apart for the works of Milani & Knežević (1992, 1993), much less attention has been paid to the study of secular resonances of higher order (nonlinear secular resonances).

It has been long believed that the effect of nonlinear secular resonances on the dynamical mobility of asteroids could be neglected when compared with chaotic diffusion in two- and three-body mean-motion resonances. Previous studies (Michtchenko et al. 2002; Lazzaro et al. 2003) showed that chaotic diffusion in nonlinear secular resonances was simply too slow to produce significant dispersion of simulated members of asteroid families, even over timescales of 2 Gyr or more.

Evolution in nonlinear secular resonances can be much faster when the Yarkovsky effect is taken into consideration. The Yarkovsky effect is a thermal radiation force that causes objects to undergo semimajor axis drift as a function of their spin, orbital, and material properties (Bottke et al. 2001). It is essentially caused by the thermal re-emission of light by the asteroids, and is present in diurnal (which can be dissipative, i.e., move asteroids towards smaller semimajor axis for retrograde rotators, and anti-dissipative, for prograde rotators) and seasonal (always dissipative) versions.

Bottke et al. (2001) showed that the unusual shape in the proper $a - e$ space of the Koronis family could be explained by evolution of family members in the $g + 2g_5 - 3g_6$ nonlinear secular resonances via the Yarkovsky effect. Vokrouhlický et al. (2002) suggested that evolution inside the $g - g_6 + s - s_6$ (named z_1 following Milani & Knežević 1993) secular resonance could explain the presence of K-type asteroids associated with the Eos family outside its boundaries. Our unpublished results for the Maria family also suggest the importance of diffusion in nonlinear secular resonances when the Yarkovsky effect is considered.

Since several V-type asteroids outside the Vesta family are in a region where previous studies of Milani & Knežević (1993) have identified several nonlinear secular resonances able to cause significant long-term variations in eccentricity and inclination, we believe that evolution in these secular resonances combined with Yarkovsky effect could have played a role in their dynamical evolution.

In this work we try to understand how the interplay of the two effects could have caused the diffusion of V-type asteroids outside the boundaries of the Vesta family. This work is so divided: in the second section we consider the general dynamics in the region of the Vesta family, in Sect. 3 we concentrate on the case of three asteroids in the $2(g - g_6) + s - s_6$ (named z_2

Table 1. Secular resonances involving the perihelion, the node, and combinations of perihelion and node of the asteroid. The first, third, and fifth columns report the resonant argument in terms of the planetary frequencies, and the second, fourth, and sixth we give the names we use in this article.

Perihelion res.	Names	Node res.	Names	Perihelion and node res.	Names
$g - 2g_5 - g_6 + 2g_7$	γ_1	$s - s_6 + g_6 - g_7$	σ_1	$2g - g_6 - g_4 + s - s_4$	ψ_1
$2g - 2g_6 + g_5 - g_4$	γ_2	$s - s_6 - g_5 + g_6$	σ_2	$2g - g_6 - g_7 + s - s_7$	ψ_2
$2g - 2g_6 + g_7 - g_4$	γ_{3a}	$s - 2s_6 + s_7$	σ_3	$2g - g_5 - g_6 + s - s_7$	ψ_3
$2g - 3g_6 + g_4$	γ_{3b}	$s - 2s_6 + s_4$	σ_4	$2(g - g_6) + s - s_6$	ψ_{4,\bar{z}_2}
$g + 2g_5 - 3g_4$	γ_4	$s - s_6 - g_7 + g_4$	σ_{5a}	$g - g_4 + 2s - 2s_6$	ψ_5
$g + g_5 + g_7 - 3g_4$	γ_{5a}	$s - s_6 + g_6 - g_4$	σ_{5b}	$g - g_5 + 2s - s_6 - s_4$	ψ_6
$g + g_5 - g_6 - g_4$	γ_{5b}	$s - s_4 - g_5 + g_6$	σ_6	$g - g_7 + 2s - s_6 - s_4$	ψ_7
$g + g_5 - 2g_6 - g_7 + g_4$	γ_{5c}	$s - s_4 + g_6 - g_7$	σ_7	$g - g_5 + 2s - 2s_6$	ψ_8
$g + 2g_7 - 3g_4$	γ_{6a}			$g - g_7 + 2s - 2s_6$	ψ_9
$g - g_6 + g_7 - g_4$	γ_{6b}				
$g - 2g_6 + g_4$	γ_{6c}				

after Milani & Knežević 1993)¹ secular resonance and their dynamical evolution when the Yarkovsky effect is considered. In Sect. 4 we study mechanisms of migration to the z_2 secular resonance from the Vesta family. In Sect. 5 we present our conclusions and we try to identify other asteroids that could have migrated from the Vesta family.

2. Secular resonances in the Vesta family region

The region occupied by the Vesta family is crossed by a very rich and complex web of secular resonances. To identify the main nonlinear secular resonances in the region we integrated a grid of 3131 particles². We computed the g and s frequencies of the particles and determined all the secular resonances up to order six. Table 1 reports a list of the principal nonlinear secular resonances of perihelion, node, and both perihelion and node, respectively, that we found with this method, with the name given in this paper. Figure 2 displays the locations of the non linear secular resonances in the osculating $a - e$ plane determined with the Spectral Analysis Method (SAM hereafter, Michtchenko et al. 2002). For reference, we also show the location (in proper elements space) of Vesta, the family members, and the 28 V-type asteroids outside the family currently known in the inner belt.

Several V-type asteroids seem to be in or very close to secular resonances in Fig. 2, and the reader might be misled to believe that most of them are currently being affected by such resonances. This is actually far from being true. In Fig. 2 we show the location of the secular resonance in the osculating $a - e$ plane, while the V-type asteroids (shown only for illustrative purposes) are in proper element space. The actual positions of the resonances are of course different in proper element space. Another problem with Fig. 2 is that secular resonances can be completely identified only in the three-dimensional

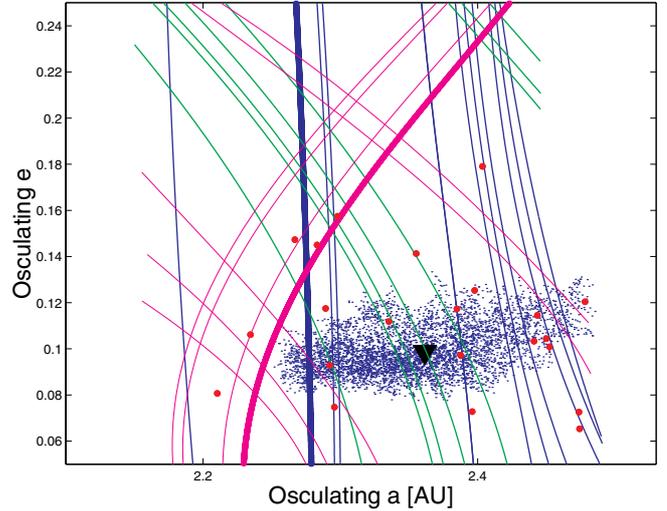


Fig. 2. Location of the nonlinear secular resonances in the 4 Vesta region, computed for the inclination of 4 Vesta ($i = 6.357^\circ$), in the osculating $a - e$ plane, determined with the Spectral Analysis Method (SAM hereafter, Michtchenko et al. 2002). In blue we show the resonances of perihelia, in green those of nodes, and in magenta those of both perihelia and nodes. For reference we also show the locations in proper element space of 4 Vesta (triangle) the Vesta family members (small dots) and of the V-type asteroids outside the family (full dots). The thick lines display the location of the z_2 (in blue) and γ_2 (in magenta) resonances. For the identification of the other nonlinear secular resonances see Table 1).

proper (a, e, i) space. The location of secular resonance in the $a - e$ plane depends on the inclination for which it was computed (Milani & Knežević 1993). Since each V-type asteroid has a different inclination, the location of the web of secular resonance in the $a - e$ plane is unique for each asteroid.

To understand whether any of the asteroid is involved (or close to) a secular resonance, we integrated the 22 V-type asteroids outside the Vesta family evolving under the influence of the planets, from Venus to Neptune over six million years (the mass of Mercury was added to that of the Sun; hereafter

¹ A z_k resonance is a resonant combination of the form $k(g - g_6) + s - s_6$.

² 101 initial values of a in the range 2.1–2.5, and 31 initial values of e , between 0–0.3, with the other elements the same as Vesta's.

Table 2. List of the 22 V-type asteroids currently known outside the Vesta dynamical family. We report number, name, proper a , e , i , absolute magnitude H , if they are in (R, which stands for resonant) or close (C, for Circulating) to a nonlinear secular resonance (see codes in Table 1), and, if they are close to one of the resonances, what is the period of circulation of the resonant argument.

Ast. #	Ast. name	a	e	i	H	Sec. res.	Circulation period
809	Lundia	2.28311	0.1450	6.7363	12.10	z_2	R
956	Elisa	2.29811	0.1575	6.4076	12.03	z_2, γ_2	R, C > 4 Myr
2113	Ehrdni	2.47377	0.0726	6.0790	12.66		
2442	Corbett	2.38765	0.0972	5.4225	12.54	σ_7	C > 1 Myr
2566	Kirghizia	2.45003	0.1044	4.4219	12.20	$\sigma_7, 9J-7S-2A$	C > 4 Myr, C > 0.05 Myr
2579	Spartacus	2.21037	0.0807	5.8947	13.03		
2640	Hallstrom	2.39777	0.1253	6.5286	12.85	4J-2S-1A, σ_6, σ_7	R, C > 4 Myr, C > 4 Myr,
2653	Principia	2.44350	0.1145	5.0773	12.24		
2704	Julian Loewe	2.38485	0.1173	5.0888	12.75	σ_7	C > 1 Myr
2763	Jeans	2.40355	0.1790	4.3185	11.99		
2795	Lepage	2.29574	0.0747	6.5979	12.75	γ_2	C > 4 Myr
2851	Harbin	2.47819	0.1204	7.7933	11.98	γ_{5a}	C > 3 Myr
2912	Lapalma	2.28927	0.1175	6.7421	12.45	γ_7	C > 1 Myr
3849	Incidentia	2.47422	0.0653	5.3938	12.63	γ_{5b}, γ_{6b}	C > 5 Myr, C > 5 Myr
3850	Peltier	2.23457	0.1061	4.7323	13.46	σ_4	C > 4 Myr
3869	Norton	2.45241	0.1009	5.2787	12.54		
4188	Kitezh	2.33538	0.1118	5.6413	12.67	σ_{5a}	C > 3 Myr
4278	Harvey	2.26689	0.1473	5.2499	13.62	ψ_2, σ_{5b}	C > 2 Myr, C > 6 Myr
4434	Nikulín	2.44110	0.1033	5.5261	12.88	γ_{6c}	C > 2 Myr
4796	Lewis	2.35523	0.1413	3.0955	13.22	σ_6, σ_7	C > 4 Myr, C > 4 Myr
4977	Rauthgundis	2.29229	0.0928	5.9408	13.99	γ_2	C > 2 Myr
5379	Abehiroshi	2.39599	0.0728	3.3537	12.55	γ_{5c}, γ_{6c}	C > 4 Myr, C > 4 Myr

this will be our standard Solar System model, unless otherwise specified), and computed the resonant argument of each nonlinear secular resonance for each asteroid. Table 2 reports the 22 V-type asteroids names and numbers, proper a , e , i , absolute magnitude H , and if they are in or close to a nonlinear secular resonance (in this case we report the approximate circulation period of the resonant angle).

Among all the detected secular resonances, the z_2 and the $2g - 2g_6 + g_5 - g_4$ (γ_2 hereafter) seem to be the most interesting, since they are the ones that are affecting the largest number of asteroids. Two asteroids are currently inside the z_2 secular resonance (809 Lundia and 956 Elisa), and three asteroids are very close to the γ_2 resonance (4977 Rauthgundis, 956 Elisa, and 2795 Lepage). As pointed out by Milani & Knežević (1993), the z_2 resonance, being a secondary mode of the $g - g_6$ resonance, is one of the strongest nonlinear secular resonance in the inner belt. The $2g - 2g_6 + g_5 - g_4$ is also expected to be a strong resonance, since it contains the combination $2g_6 - g_5$, which is the leading critical term of the secular planetary theory (Nobili et al. 1989).

In view of this, it would be necessary to determine the location in the osculating $a - e$ space of these two resonances, in particular in the region of 956 Elisa, which is the asteroid currently inside the z_2 and very close to the γ_2 resonances. This is not as simple a task as it may seem. Contrary to the case of the most important linear secular resonances ν_6 , ν_5 , and ν_{16} ,

whose dynamics has been studied in great detail by Morbidelli & Henrard (1991a,b), no analytical model has ever been made for nonlinear secular resonances. To study orbits inside these nonlinear resonances we cannot use the approach of Migliorini et al. (1997) for the ν_6 resonance, where a grid of test particles in initial osculating $a - e$ was created and the inclination was determined by the requirement of being at the exact resonance. We need to find an alternative procedure to map the new resonances.

To determine the location of a nonlinear secular resonance there are essentially three methods: i) use the second order and fourth-degree secular perturbation theory of Milani & Knežević (1990) to find the g and s precession rates of the longitudes of perihelion and node of asteroids, and, therefore, the locations where such values are close to a resonance in proper element space; ii) compute the frequencies numerically, analyzing the equinoctial elements ($e \cos \varpi, e \sin \varpi$), ($\sin(I/2) \cos \Omega, \sin(I/2) \sin \Omega$) of test particles to obtain g and s with the frequency modified Fourier transform (FMFT, hereafter) of Šidlichovský & Nesvorný (1997) or with SAM and, therefore, the location of the resonances in osculating space; or iii) plot the resonant argument of the secular resonances and define the boundary as the place where we pass from circulation to libration.

The Milani & Knežević approach, while very useful to obtain a qualitative understanding of secular resonances' dynam-

ics, has some drawbacks when you are interested in precisely determining the location of secular resonances in a numerical simulation. First, the Milani & Knežević method does not include the effect of the perturbations of the terrestrial planets, that are important for the dynamics in the Vesta family region. Also, it is an analytical model based on a linear theory, so the values of the asteroid and planetary frequencies (and therefore the locations of the secular resonances) are usually different from those computed with a numerical simulation. Since the actual location of the secular resonances is essentially unique to each different simulation (for example, neglecting the indirect effects of the perturbations of the terrestrial planets on the jovian ones may change the planets frequencies by up to $0.05''/\text{yr}$), in this work we prefer to use a numerical technique.

Of the two numerical methods, determining the resonances by computing the frequencies of the particles presents two disadvantages. To apply this method we need to numerically determine the values of the proper frequencies. Small errors in the computation of g_4 , for example, may lead to an erroneous determination of the $2g - 2g_6 + g_5 - g_4$ (γ_2) resonance. Moreover, this method does not give any information on the resonance width. For these reasons, we decided to use only the last method, i.e., plotting the resonant argument.

To map the z_2 and γ_2 resonances we generated a grid of 1250 test particles, with 50 initial values of a values from 2.29360 to 2.31810 AU, and 25 initial values of e , ranging from 0.12 to 0.21, and all the other angles the same as 956 Elisa. We integrated them over 6 Myr, enough to sample at least one libration period of both resonant arguments, plotted the resonant argument of each test particle, and determined where it passed from circulation to libration. Figure 3 shows the locations of the two resonances in the initial osculating $a - e$ plane.

So far, we have determined the sections of the two resonances in the initial osculating $a - e$ plane. To completely study the resonances, we should have done two other series of simulations, so as to obtain the resonances' sections in the $a - i$ and $e - i$ planes, and this for each of the 28 V-type asteroids outside the Vesta family! Rather than following this computationally expensive approach, we decided to study the dynamics of asteroids inside the z_2 resonance by integrating clones of the three resonant asteroids under the influence of the Yarkovsky force. This approach not only could tell us if the Yarkovsky force may take the asteroids out of the z_2 resonance, but also give us precious hints on the asteroids' origins. We will discuss the results of this approach in more detail in the next section.

3. Evolution in the z_2 secular resonance under the Yarkovsky effect

In the previous section we identified a web of secular resonances in the region of the Vesta family, and determined which asteroids are in or close to a nonlinear secular resonance. It turns out that only two asteroids are currently in a nonlinear secular resonance: 809 Lundia and 956 Elisa, which are inside the z_2 resonance (see Table 3).

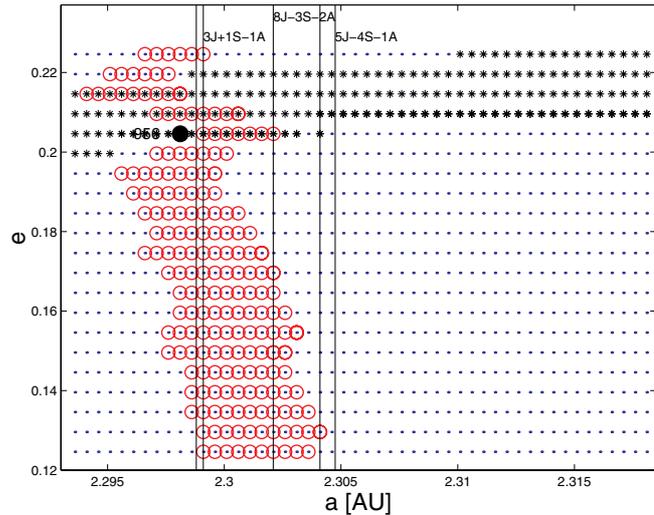


Fig. 3. The plot shows the location of the z_2 ($2(g - g_6) + s - s_6$) secular resonance (black asterisks), of the γ_2 ($2g - 2g_6 + g_5 - g_4$) (red circles), and of the 3J+1S-1A, 8J-3S-2A, and 5J-4S-1A three-body resonances (vertical lines), as found by plotting the resonant arguments of the test particles. The black full circle displays the actual position of 956 Elisa, and small dots show all the initial conditions used. The 3J+1S-1A resonance lies in the band from 2.29880 to 2.29910 AU (as compared with the analytical result of Nesvorný & Morbidelli 1998, of 2.2994 AU).

In this work we are interested in the origin of V-type asteroids outside the Vesta dynamical family. Usually, studies on Yarkovsky diffusion of asteroid families use the approach of numerically integrating test particles simulating the orbital initial conditions of the family members immediately after the family formation. These test particles are evolved under the influence of the Yarkovsky effect and allowed to interact with the local web of resonances (mean-motion and secular). The analysis of their evolution provides hints on their possible paths outside the dynamical families (Bottke et al. 2001).

This approach has both merits and drawbacks. We have seen that nonlinear secular resonances can be very narrow in osculating elements space. Therefore, by integrating clones with random initial osculating elements, their interaction with a specific secular resonance might be overlooked. Moreover, in this case we are in a rather different situation than the typical one. We not only know where possible former family members are presently located, but we also know that some of these bodies are currently interacting with a specific nonlinear secular resonance.

In view of these considerations, we adopted a different approach. We chose to integrate clones of the two asteroids currently inside the z_2 resonance with the Yarkovsky effect. Since the Yarkovsky effect depends on asteroid size (smaller objects drift faster), we considered clones with the same initial orbital parameters of the original asteroids, but with different diameters. From their dynamical evolution, we try to infer the possible paths that might have brought them to their current orbital position with respect to the Vesta family. This approach not only can give hints at the past dynamical history of the two as-

Table 3. The 33 asteroids close in proper element space to 809 Lundia, and 956 Elisa, identified with the hierarchical clustering method described in Sect. 4. We report the asteroid identification, the proper a , e , $\sin(i)$, if the asteroid is inside the z_2 secular resonance, and the spectral type (if known).

Ast. #	a	e	$\sin(i)$	z_2 res.	Spect. type
956	2.29812	0.1575	0.1116	yes	V
11918	2.3106	0.1566	0.114	no	
24840	2.30014	0.1604	0.1131	yes	
7589	2.28564	0.16	0.1135	yes	
3083	2.28444	0.1565	0.1168	no	
14948	2.27722	0.1597	0.1107	yes	
16556	2.28112	0.1644	0.1129	no	
17251	2.287	0.1532	0.1132	yes	
5238	2.27456	0.1562	0.1105	yes	
22351	2.27637	0.1574	0.1095	no	
6481	2.27887	0.1639	0.1161	no	
20070	2.27245	0.165	0.1099	no	
25343	2.2746	0.1624	0.1162	no	
16377	2.29084	0.1529	0.1136	yes	
24755	2.29664	0.1507	0.1156	yes	
809	2.28311	0.145	0.1173	yes	V
4278	2.26689	0.1473	0.0915	no	V
21829	2.27416	0.1437	0.1171	yes	
12227	2.27266	0.1424	0.114	yes	
18082	2.26536	0.1454	0.1163	yes	
21935	2.27521	0.1416	0.1179	yes	
22598	2.27857	0.1417	0.1128	yes	
19373	2.26626	0.1484	0.1146	no	
19738	2.28213	0.1415	0.121	yes	
1476	2.28112	0.1445	0.1124	yes	
24918	2.28185	0.1398	0.1151	yes	
20181	2.28277	0.1386	0.1222	yes?	
17453	2.28957	0.1455	0.1118	yes	
19672	2.28517	0.139	0.1168	yes?	
12671	2.29168	0.1382	0.1198	no	
14191	2.29302	0.1456	0.1089	no	
7550	2.29017	0.1472	0.1079	no	
16740	2.28944	0.1457	0.1066	no	
24303	2.29396	0.1449	0.1062	no	
14444	2.27964	0.1455	0.1077	yes	

teroids, but can tell us for which sizes the Yarkovsky drift can drive the bodies out of the z_2 resonance.

To simulate the Yarkovsky effect we used SWIFT-RMVSU, the version of SWIFT modified by Brož (1999) to account for both versions of the Yarkovsky effect. We created 10 clones each with the same orbital elements of 809 Lundia, 956 Elisa, and 6406 1992 MJ, with radii of 100 m, 500 m, 1 km, 2 km, 3 km, 4 km, 5 km, 6 km, 7 km, and 8 km (the maximum possible size of the two asteroids) and integrated them with a step

size of 20 days. We referred all elements with respect to the invariable plane of the Solar System, and used the following thermal parameters: $\rho_{\text{bulk}} = 3500 \text{ kg/m}^3$, $\rho_{\text{surface}} = 1500 \text{ kg/m}^3$, $K = 2.65 \text{ W/m}^3/\text{K}$, $C = 680 \text{ J/Kg/K}$, $A = 0.1$, and $\epsilon = 0.9$ (Farinella et al. 1998).

All asteroids were either prograde rotators with 0° obliquity, (so that the diurnal Yarkovsky effect was anti-dissipative, and the semimajor axis evolved towards larger values), or retrograde rotators (evolution towards smaller a), with 180° obliquity. The period of rotation was assumed proportional to the inverse of the radius as in Farinella et al. (1998). Since our goal was to study diffusion in the z_2 secular resonance, we did not consider reorientations of spin axis via collisions or YORP (Čapek & Vokrouhlický 2004). Our simulated asteroids have constant obliquity and rotation periods. This assumption will not give realistic results for the timescale of dynamical evolution of asteroids, but will allow them to more efficiently evolve in the z_2 secular resonance³.

All asteroids under study are in a region that is roughly delimited by two strong mean-motion resonances: the 7J-2A and the 5J-4S-1A (roughly speaking, a going from 2.255 to 2.304 AU). In our work we will concentrate only on this region of space. We integrated the asteroids for 1.5 Gyr, a time that was sufficient to allow the 100 m clones to cover the region between the two mean-motion resonances. By integrating smaller asteroids than the actual ones inside the z_2 resonance we could obtain a faster Yarkovsky drift and study diffusion in the resonance on timescales much shorter than those required for km-sized objects.

We filtered the osculating orbital elements (Carpino et al. 1987) so as to eliminate all frequencies with periods smaller than 1333 yr, and computed synthetic proper elements (Knežević & Milani 2000) for each ≈ 10 Myr interval⁴. We also computed averaged elements with a running window of 10 Myr and a shift of 1 Myr (Nesvorný & Morbidelli 1998). While averaged elements are not constants of the motion like proper elements, they give qualitative information on the dynamical evolution of asteroids and are easier to compute. Our results show that apart from a shift of ≈ 0.05 in e and $\approx 0.2^\circ$ in i , the qualitative behavior is exactly the same either for averaged or proper elements. For simplicity, we will only show averaged elements in our figures.

We start by showing the results for 956 Elisa because the dynamics nearby is particularly interesting. As seen in Fig. 3, 956 Elisa is near two three-body resonances and a nonlinear secular resonance. In the next section we will show how members of the dynamical family could migrate to the z_2 by capture

³ We should point out that our assumption of zero obliquities may not be that naive. Studies of Čapek & Vokrouhlický (2004) show that when thermal conductivity is considered, asteroids do tend to evolve towards zero obliquities (and progressively slower or faster spins rates).

⁴ The proper semimajor axis was computed by averaging the filtered elements over 10 Myr. The synthetic eccentricity and inclination were determined analyzing the equinoctial elements ($e \cos \varpi$, $e \sin \varpi$; $\sin \frac{1}{2} \cos \Omega$, $\sin \frac{1}{2} \sin \Omega$) with FMFT. The amplitudes associated with the largest frequencies were the synthetic proper e and i .

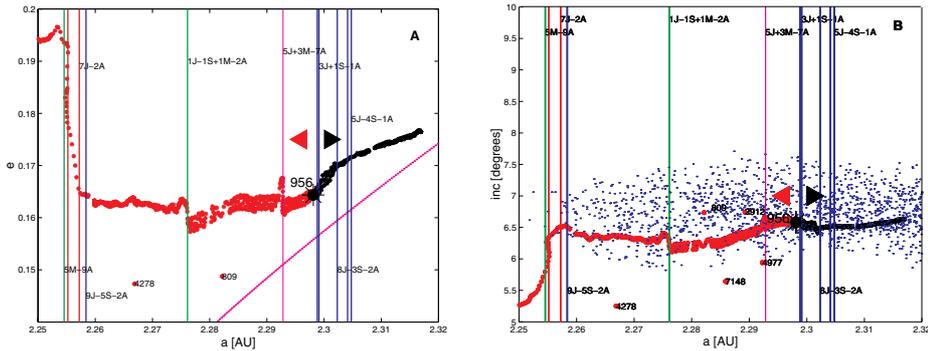


Fig. 4. The averaged $a - e$ and $a - i$ evolution of two 100 m clones of 956 Elisa. In black we show the evolution of the prograde clone, in red of the retrograde one. The arrows show the two directions of evolution in a , starting from 956 Elisa (black full dot). Small dots show the orbital location of members of the Vesta dynamical family. The magenta line in Fig. 4a gives the location of the z_2 secular resonance, computed for the inclination of Vesta (see Fig. 2) and is given for reference. Note how the clone of Elisa crosses the three-body resonances (3J+1S-1A and 5J-4S-1A, where hereafter J stands for Jupiter, S for Saturn, M for Mars, U for Uranus, and A for asteroid), while remaining inside the z_2 resonance (its path in the $a - e$ plane is inclined, while it should be a horizontal line if the particle were freely drifting due to Yarkovsky effect). The clone leaves the resonance only when interacting with the 1J-1S+1M-2A four-body resonance. It then freely drifts due to the Yarkovsky effect until it reaches the 7J-2A resonance where it is temporary captured.

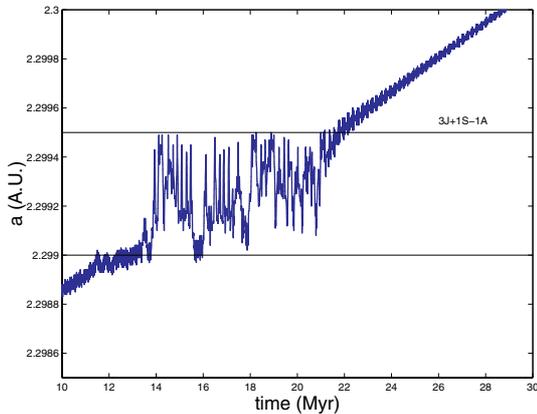


Fig. 5. Evolution of the osculating semimajor when the 100 m clone of 956 Elisa crossed the 3J+1S-1A mean motion resonance.

in one of these resonances via Yarkovsky effect. The simulations of the 100 m clone, however, already offers interesting results. Figures 4 show the averaged $a - e$ and $a - i$ evolution of two 100 m clones of Elisa (averaged elements computed over a period of 10 Myr). In red we show the diffusion path of the retrograde clone, and in black that of the prograde clone. The arrows display the directions of propagation. Note how the prograde clone stays inside the z_2 resonance when crossing the 3J+1S-1A, the 8J-3S-2A, and the 5J-4S-1A resonances. Figure 5 shows enlargements of the evolution of the semi-major axis of Elisa when crossing the 3J+1S-1A resonances.

The most important fact is that the clone remains inside the z_2 secular resonance, until it reaches the 1J-1S+1M-2A four-body resonance. It then escapes from the secular resonance and drifts freely due to the Yarkovsky effect, until it reaches the 7J-2A resonance. The particle leaves the secular resonance in the region between the 5J-4S-1A and the 7J-2A resonances: this could explain the existence of asteroids like 4278 Harvey, that are in this region but not in a secular resonance. After crossing the 8J-3S-2A mean motion resonance,

the argument of resonance changes libration amplitude, as can be seen in Fig. 6a.

On the right panel of Fig. 6, we show the time behavior of the z_2 resonant argument when the particle crossed the 1J-1S+1M-2A four-body resonance. The fact that the simulated asteroid stayed inside the z_2 resonance even when this was crossed by powerful three-body resonances strongly suggest that the z_2 could be an efficient mechanism to capture objects running away from the Vesta family in three-body (such the 5J-4S-1A, or the 3J+1S-1A), or in nonlinear secular resonances (such as the γ_2). We will further investigate this hypothesis in the next section.

The case of the clone of 809 Lundia is easier to interpret. The clone essentially followed the secular resonance z_2 in the whole region between the 5J-4S-1A and the 7J-2A resonances (Fig. 7). In the region near the 7J-2A resonance, the z_2 resonance interacts with other secular resonances. Once the clone reaches the 7J-2A it escaped from the secular resonance.

Finally, we want to see what happens for larger asteroids. Capture in mean-motion resonance is a process that strongly depends on the rate of migration of the asteroid which is a function of the asteroid's diameter. We want to understand for what size capture is possible.

We start by analyzing the evolution of clones of Lundia. Since the region around Lundia is relatively free of resonances, all the clones essentially drift freely inside the z_2 secular resonance, at a rate that depends on the asteroid's size. We may ask ourselves if diffusion in nonlinear secular resonances due to Yarkovsky effect could be as effective as diffusion in free space. In other words, is the Yarkovsky diffusion slowed down by the fact that particles are in a secular resonance, or does the diffusion act at same speed, but with a different geometry (following an inclined path in the $a - e - i$ space instead of a horizontal line of constant e and i)? To answer this question, we computed how much particles inside the z_2 secular resonance diffused in proper element space, under the influence of the other planets from Venus to Neptune, or under the influence of the Sun only.

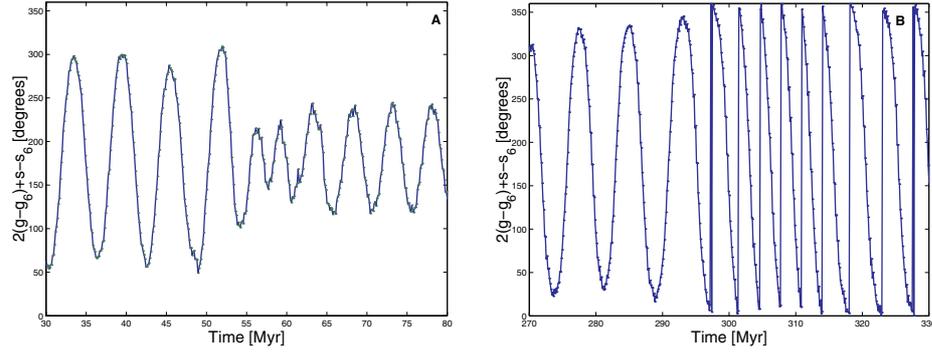


Fig. 6. The argument of the z_2 resonance for the two 100 m Elisa clones evolving under the Yarkovsky effect. **a)** shows the resonant argument for the clone evolving towards larger a , when passing through the 8J-3S-2A mean-motion resonance. **b)** shows the argument of the clone evolving towards smaller a , when crossing the 1J-1S+1M-2A four-body resonance (in this case the clone leaves the z_2 secular resonance). The resonant angles have been processed through a digital filter in order to remove all frequencies with periods smaller than 300 000 yr.

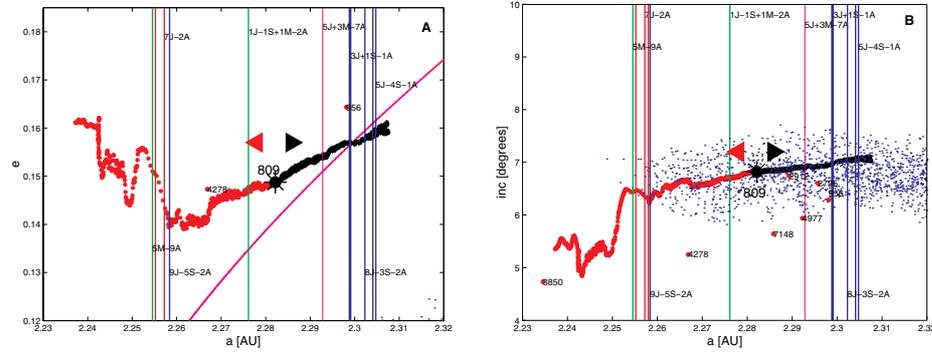


Fig. 7. The proper $a - e$ and $a - i$ evolution of two 100 m clones of 809 Lundia. In black we show the averaged elements of the prograde clone and in red those of the retrograde one. The arrows show the directions of propagation. In the region between the 7J-2A and the 5J-4S-1A the clone never leaves the z_2 secular resonance.

To compute the distance covered in proper element space, we used the metric from Zappalà et al. (1990):

$$\delta v = na \sqrt{\frac{5\delta a^2}{4a^2} + 2\delta e^2 + 2\delta \sin^2 i}. \quad (1)$$

Where v is the distance (in m/s) and n is the asteroid's proper motion. We then computed the equivalent distance covered in semimajor axis. The thermal parameters are the usual for V-type asteroids. In all cases we considered, the distance covered in the secular case is always larger than that for the free case. Changing the metric does not change these results by more than 5%. This suggests that diffusion inside the z_2 secular resonance via Yarkovsky effect is at least as fast as diffusion via Yarkovsky effect in free space.

The case of Elisa is more interesting. The clones of Elisa passed through the 3J+1S-1A, the 5J-4S-1A, and the γ_2 resonances while remaining inside the z_2 for asteroid radii up to 2000 m. Capture (and temporary capture) into the 3J+1S-1A, the 8J-3S-2A and the γ_2 was possible only for larger radii. Figure 8 shows the evolution in the $a - e$ space of a 4000 m clone of Elisa that was finally captured into the γ_2 secular resonance. The fact that only particles larger than ≈ 4 km in diameter are captured into one of the weaker three-body or other secular resonances has interesting repercussions on the current population of asteroids inside the z_2 , that will be investigated in more detail in Sect. 4.1.

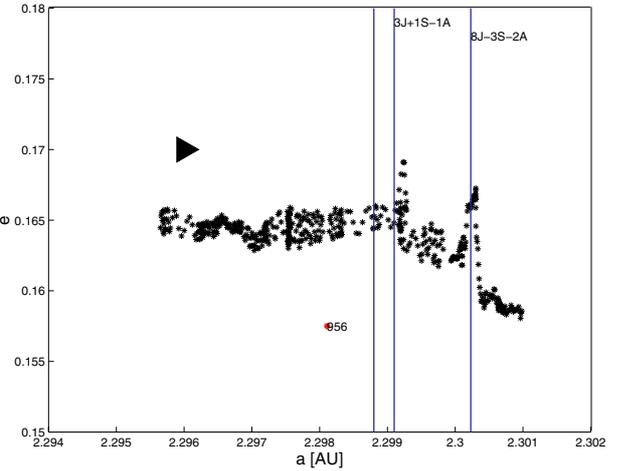


Fig. 8. Averaged $a - e$ evolution of a 4000 m clone of Elisa. The particle is temporarily captured into the 3J+1S-1A three-body resonance, passes through the 8J-3S-2A resonance, and is finally captured into the γ_2 secular resonance. The black arrow displays the direction of propagation.

4. Mechanisms of migration from the Vesta family to the z_2 secular resonance

Simulations with clones of 956 Elisa have shown that asteroids initially inside the z_2 secular resonance and evolving un-

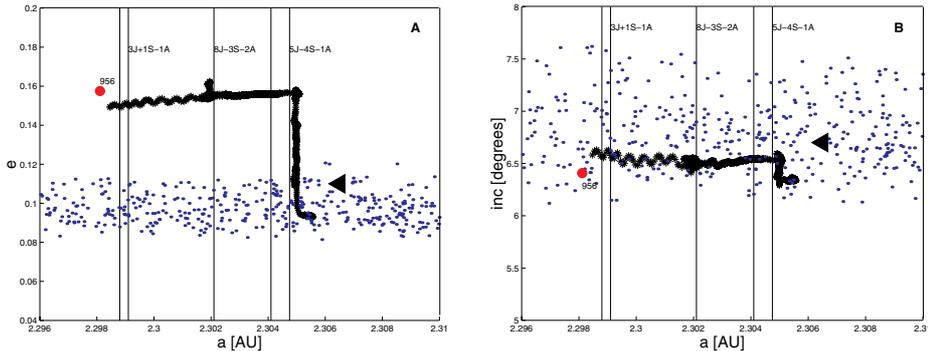


Fig. 9. Averaged $a - e$ **a)**, and $a - i$ **b)** evolutions of a 8000 m radius asteroid near the 5J-4S-1A three-body resonance under the effect of the Yarkovsky force. The black arrows show the directions of propagation. The full dot shows the location (in proper element space) of 956 Elisa, while the small dots identify Vesta-family members. Only asteroids with $D > 8$ km can be captured in the three-body resonance and remain inside it for time long enough to reach the transition region with the z_2 resonance. Once there, the particle was captured in the z_2 resonance, and drifted until reaching values of a , e , and i comparable to those of 956 Elisa.

der the Yarkovsky effect tend to stay inside the resonance when crossing three-body or other secular resonances and only escape when the resonance crosses a relatively strong two-body resonance like the 7J-2A. This suggests a possible evolution scheme for some Vesta family members: asteroids are initially captured inside three-body resonances like the 5J-4S-1A or the 3J+1S-1A, or in other secular resonances (such as the γ_2), where they drift until reaching the zone where these resonances cross the z_2 resonance. Then, at least a fraction of them is captured into the resonance where they slowly drift until reaching the 7J-2A mean motion resonance.

In the next subsections we will study in more detail these mechanisms of migration, and we will try to address why asteroids currently in the z_2 resonance have larger diameters than the typical Vesta family member.

4.1. Three-body resonances

In this section we concentrate on the 5J-4S-1 resonance because it is a zero-order resonance and previous study (Lazzaro et al. 2003) have shown that chaotic diffusion in this resonance is quite effective at increasing asteroid eccentricities to regions close to the domain of the z_2 secular resonance. We believe that our results could qualitatively be extended to the case of other three-body resonances.

We simulated members of the Vesta dynamical family by using test particles of seven different sizes (100, 1000, 4000, 5000, 6000, 7000, and 8000 m), just outside the 5J-4S-1A resonance ($a = 2.3052$ AU), for seven values of osculating e (from $e = 0.06$ to 0.12, with a step of 0.1) and the same inclination of Elisa. Since we are working with retrograde rotators of zero obliquity, our results will give an upper limit on the size of objects that can diffuse through the resonance. The actual size limit will depend on the cosine of the real obliquity.

Figure 9 shows the $a - e$ and $a - i$ evolutions of a 8 km clone integrated with SWIFT-RMVSYS (same thermal elements as in Sect. 3) for 1.2 Gyr with initial e equal to 0.09. Of the integrated particles, only those with $R > 4$ km were captured into the three-body resonance. Of the captured particles, only those with $R = 8$ km drifted along the 5J-4S-1 until they reached the

z_2 secular resonance where they evolved until reaching the orbital location of 956 Elisa. These results did not significantly depend on the initial values of e_0 .

This simulation demonstrated that it is possible to be captured into the z_2 from the 5J-4S-1 resonance, but that only relatively large bodies can stay inside the 5J-4S-1 for times long enough to reach the transition region with the z_2 resonance. Smaller bodies are either drifting too fast to be captured into the 5J-4S-1A resonance or, even if captured, do not remain inside this resonance long enough to reach the domain of the z_2 resonance. Moreover, if we assume that 956 Elisa is a runaway fragment of the original Vesta family, this simulation provides an estimate on the age of the Vesta family, which should be at least as old as the time required for Elisa to migrate to its current position (and possibly older, since we neglected reorientations, and the evolution in the three-body and secular resonances was the fastest possible).

To further study how the capture mechanism depends on the asteroid size, we integrated 33 particles in the semimajor axis range between 2.30360 and 2.30520 AU, with the other orbital elements of 956 Elisa (the 5J-4S-1A resonance is limited to the interval 2.30410-2.30475 AU). We made three runs with particles of 100 m, 4000 m, 8000 m of radii and with the usual thermal parameters. We want to understand if capture into the z_2 depends on the asteroid size, and, since we now know that migration in the 5J-4S-1A is possible, we started with particles as close as possible to the transition region of the 5J-4S-1A with the z_2 resonance.

Of the integrated particles inside the 5J-4S-1A resonance, 58% of the 100 m, 55% of the 4000 m, and 64% of the 8000 m particles were captured into the z_2 . This shows that capture from the three-body resonance into the z_2 is not strongly dependent on the asteroid size. The size-dependent selection process, in our opinion, operates in the region where the three-body resonances cross the Vesta dynamical region. In our scenario only relatively large asteroids are drifting slow enough via Yarkovsky effect to be captured into the 5J-4S-1A resonance, where they evolve until reaching the z_2 resonance, and are finally captured. This scenario could explain why the three asteroids in the z_2 resonance all have estimated diameters

much larger than the average diameter of Vesta family members (>10 km, as opposed to ≈ 4 km).

4.2. Other nonlinear secular resonances: the γ_2 resonance

In Sect. 3 we showed that asteroids currently inside the z_2 resonance can interact with other secular resonances when the Yarkovsky effect is considered. For example, a 4000 m clone of 956 Elisa was captured in the γ_2 secular resonance. Here we try to understand if a Vesta-family asteroid migrating via the Yarkovsky effect could be captured into one of the secular resonances and then drift until it meets the z_2 . Since our study focuses on the dynamical evolution of 956 Elisa, we concentrate on the γ_2 resonance. Once again, we believe that our results may be qualitatively extended to the cases of other nonlinear resonances.

We started by simulating fictitious members of the Vesta family already inside the γ_2 resonance, and evolving under the influence of the Yarkovsky effect. We used 33 clones of 956 Elisa (see Fig. 3) with semimajor axis ranging from 2.29360 to 2.30960 AU, with e equal to 0.19960 and the other elements the same as 956 Elisa. Of these test particles, 10 were in resonance and 10 more in quasi-resonance. We made three integrations with particles with radii of 100 m, 4000 m, and 8000 m, under the influence of the Yarkovsky force (we used the usual thermal parameters of V-type asteroids).

Of our simulated particles, the smaller ones with 100 m radius left the γ_2 resonance as soon the simulation started. Of the larger particles, only a test particle of 8000 m radius stayed inside the γ_2 over the 300 Myr of the integration (the integration time, however, was not long enough to allow the particle to reach the transition region with the z_2 resonance). While it is possible that particles captured in the γ_2 resonance could migrate until they reach the z_2 resonance, our simulations suggest that this mechanism is much less effective than transport via a three-body resonance.

Once test particles reach the transition region, is capture into the z_2 resonance a size-dependent process? To answer this question, we integrated 33 clones of 956 Elisa with semimajor axes going from 2.29360 to 2.30960, an eccentricity value of 0.12460, and all the other elements the same as 956 Elisa, under the effect of the Yarkovsky force. Seven of these clones were initially in the γ_2 resonance with the other 8 in semi-resonance. We made a series of three simulations with particles of 100, 4000, and 8000 m in radius, and the results were always the same: except for the occasional particle interacting with a three-body resonance, all of the simulated asteroids were captured into the z_2 resonance once they reached the transition region. Once again, the process of capture into the z_2 resonance seems not to be a size-dependent process.

4.3. The 7J-2A mean-motion resonance

We saw in Sect. 3 that the clones of the resonant asteroids evolved inside the z_2 until reaching the 7J-2A mean-motion

resonance. After passing through the 7J-2A resonance, none of the integrated particles remained in the z_2 secular resonance. To understand if it is possible to pass from the 7J-2A to the z_2 , we integrated 28 retrograde particles with zero obliquity with SWIFT-RMVS. The particles had seven values of osculating e (from $e = 0.06$ to 0.12, with a step of 0.1) and an initial a of 2.26013 AU. The initial values of the angles (i , Ω , ω , M) were those of 809 Lundia, and we used four values of the radii (100 m, 1000 m, 4000 m, and 8000 m), while the thermal parameters are the same as used in previous simulations. The particles drifted via the Yarkovsky effect to the separatrix of 7J-2A resonance, and then continued their evolution.

Of the integrated particles, some of them evolved in the 7J-2A resonance until reaching Mars-crossing orbits, others interacted with the 9J-5S-2A three-body resonance but, as expected, none were captured by the z_2 resonance. While our simulations do not exclude that such a path is possible, it seems to be quite improbable.

5. Discussions

In this work we studied a possible migration route from the Vesta dynamical family to the orbits of two V-type asteroids inside the z_2 nonlinear secular resonance. We found that:

1. Members of the Vesta dynamical family migrating via the Yarkovsky effect are first captured into three-body (5J-4S-1A, 3J+1S-1A, etc.), nonlinear secular resonances ($2(g - g_6) + g_5 - g_4$) and start migrating towards orbits of higher eccentricities. Only asteroids of $D > 8$ km can be captured in one of these resonances and this could explain the fact that the three V-type asteroids in the z_2 have estimated diameters larger than 10 km, which is above the average size of Vesta family members.
2. Once they reach the zone where those resonances cross the z_2 secular resonance, members of the Vesta family can be captured and start their evolution towards larger or smaller a inside the resonance. The process of capture into the z_2 resonance is essentially size-independent.
3. Since the typical times required for the clones of Elisa to migrate from the Vesta family to the current location of 956 Elisa is of at least 1.2 Gyr, if we assume that 956 Elisa was formed in the same cratering event that created the Vesta family, our simulations set a lower limit on the age of the family.
4. Asteroids generally stay inside the z_2 secular resonance until they reach relatively strong mean motion resonances like the 7:2 (or other weaker three- or four- body mean motion resonances). The residence time inside the z_2 is of the order of 1 Gyr.
5. We believe other V-type asteroids could have followed the same path and currently be in the z_2 secular resonance. We identify several possible V-type asteroids inside this resonance.

While in this paper we suggested a possible scheme for the origin of two V-type asteroids outside the Vesta dynamical family, currently 20 other similar bodies are known. The migration of

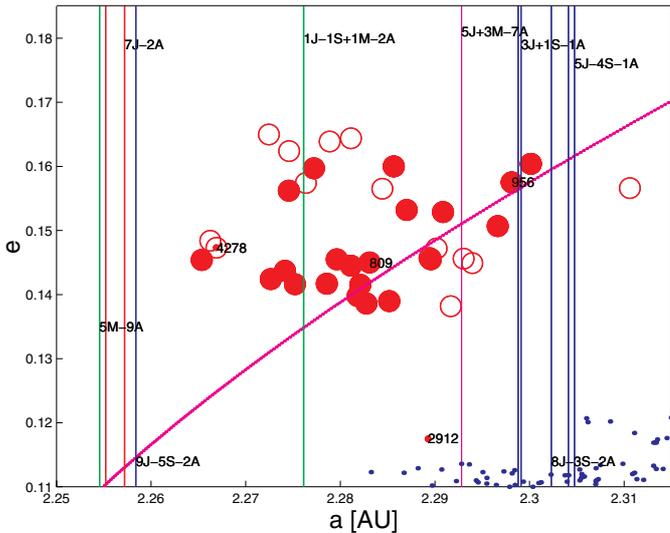


Fig. 10. Neighbors of 809 Lundia and 956 Elisa obtained with hierarchical clustering method for an absolute magnitude of 14, and a cutoff velocity of 130 m/s (Zappalà et al. 1990). The asteroids currently in the z_2 resonance are indicated by full dots, while the non-resonant ones are shown by empty dots. The diagonal line displays the location of the z_2 secular resonance, computed for the inclination of 4 Vesta, in osculating $a - e$ space. Small dots show the orbits of family members. Note how 4278 Harvey could have been found by using this method.

these bodies from the Vesta family may have involved evolution in other nonlinear secular resonances, or different mechanisms of dynamical migration, like gravitational scattering by massive asteroids (Carruba et al. 2003). We believe this is an interesting problem, that promises to offer still several challenges to dynamicists.

On the other hand, now that we know it is possible to migrate from the Vesta family to the z_2 secular resonance we may ask ourselves if other asteroids may have followed the same route. Are there any other V-type asteroids near 809 Lundia and 956 Elisa? To answer this question we searched for minor bodies close in proper element space to the three resonant asteroids. We only considered objects of absolute magnitude up to 14.5 (the current maximum magnitude of V-type asteroids outside the Vesta family in the inner belt) and with mutual distances in proper element space of less than 130 m/s. Table 3 and Fig. 10 show the asteroids we found. With the exception of 4278 Harvey (and, of course, the two resonant asteroids), that has a V-type spectrum, no spectral classification is currently available for the other bodies.

We integrated these objects for 6 Myr and verified that most of them are currently inside the z_2 resonance. In our opinion, some of these asteroids could have migrated from the Vesta family in a way similar to that of 809 Lundia and 956 Elisa. We encourage observers to obtain a classification for these bodies in order to determine if any of them belong to the V-type class.

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