The Long-term Dynamical Stability of Jupiter's Trojan Asteroids

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Running Title: Dispersal of the Trojan Swarm

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Trojan asteroids are objects that orbit the Sun with the same mean semi-major axis as Jupiter, but lead or trail the position of Jupiter in its orbit by $\sim 60°$. One very interesting aspect of the Trojan swarms is that a significant number of asteroids are on orbits that analytic theory suggests should be unstable[1]. This has lead several researchers to conclude that there is a flaw in the analytic theory. Here we present the results of the first long-term dynamical integrations of the orbits of Trojan asteroids. We find that the Trojan swarms are indeed not indefinitely stable. The gravitational effects of the giant planets have whittled away the swarms' outer boundary over time. We estimate that there are currently over 1200 escaped Trojans larger than $1\,km$ in diameter currently roaming the solar system, a few of which are on Earth-crossing orbits.

Trojans are asteroids that orbit the Sun about the L_4 and L_5 Lagrange points of Jupiter and are locked in the 1:1 mean motion resonance with Jupiter. They orbit the Sun at the same average heliocentric distance and with the same average orbital period as Jupiter. Estimates[1] indicate that the Trojan swarms contain a number of objects similar to or greater than that in the asteroid belt. Early analytic work on the three body problem (Sun-Jupiter-asteroid in this case) has shown that objects with the same semi-major axis as, but placed $\pm 60°$ from Jupiter are stable. Objects slightly displaced from these two points will oscillate about them[2]. Indeed, there is a rich literature exploring the behavior of these asteroids using analytically methods, including the works of Eŕdi[3], and Rabe[4]. Trojan asteroid orbits have also been studied by Bien & Schubart[5], Mikkola & Innanen[6], and Milani[7] and others using relatively short numerical integrations.

Most Trojans asteroids are likely the remains of a population of planetesimals that formed the core of Jupiter[1]. In addition, as they are dynamically dominated by Jupiter, understanding the dynamical structure of the Trojan swarms may lead to important clues to the origin and early dynamical evolution of this planet. Perhaps the most important aspect of the structure of the swarms is their outer boundaries. The only prior attempt to understand the outer boundary of the swarms was by Rabe[8], using analytical techniques. However, a comparison between his result and the distribution of real Trojans, Figure 1, shows that a significant number of real Trojans lie above Rabe's stability curve. (It should be noted that these Trojans were discovered after Rabe published this work.)

The discrepancy between the observations and theory prompted Shoemaker, Shoemaker & Wolfe[1] to argue that the true stability curve lies above the curve calculated by Rabe. Their argument was based on the long-held assumption by the astronomical community that the solar system and structures in it are dynamically stable. However, recent results[9,10,11] have shown that chaos, caused by the overlap of resonances within the solar system, can lead to behavior of small bodies that appears stable for billions of years after which a global instability may occur. These results call into question both the the conclusions of Shoemaker, Shoemaker & Wolfe and the nature of the outer boundary of the Jupiter's Trojan swarms.

In an attempt to understand the origin of the objects above Rabe's stability curve, we first numerically integrated the orbits of 270 fictitious massless L_4 Trojan asteroids in three dimensions under the influence of the Sun and the four giant planets; the particles themselves were not gravitationally interacting with each other. The equations of motion for the Sun, planets, and particles were integrated using the very efficient MVS integration scheme developed by Wisdom & Holman[13], with a timestep of 1/2 year. We integrated the orbits of the test particles for for 10^9 years or until they escaped the swarms.

Each of our fictitious Trojans had different initial values of libration amplitude, D, and proper eccentricity, e_p (cf. discussion in Figure 1). These initial proper elements were chosen from a grid of 27 values of e_p ranging from 0 to 0.8 and 10 values of D between 0 and 140°. In addition, associated with both D and e_p, which are amplitudes of values that oscillate as a function of time, are two phases, which were chosen to be the same for all 270 particles. However, through a preliminary set of calculations, we find that the dynamical lifetimes of particles is not significantly effected by our choice of the phases. The test particles initially started in the same plane as Jupiter, but the planets had their correct inclinations and the calculation was performed in three dimensions.

We were interested in whether an orbit is stable for the length of the integration. An orbit was deemed unstable if the particle either suffered a close approach with Jupiter and was ejected from the solar system or visited all four quadrants of a coordinate frame that rotates with the Sun-Jupiter system. If a particle visited all four quadrants of such a frame than it was either in a nonresonant or a horseshoe orbit and was no longer a member of the Trojan swarms.

The results from this initial study are shown in Figure 1 as contours of dynamical lifetime. The integrations show that the observed upper boundary in e_p–D space occupied by Jupiter Trojans is time dependent. In times past, it was significantly larger than at present and it has been slowly shrinking with time. Indeed, the area of physical space inhabited by Trojan asteroids shrinks by almost a factor of two between 10^8 and 10^9 years. Another, result from our billion year survey is that a significant number of the present Trojans lie above or to the right of our billion-year stability curve. About 10% of the Trojans lie in a region that is unstable on the billion year timescale while another 10% fall in the region between the lines marked by open and closed triangles in Figure 1.

In order to better understand and test the observation that a significant number of Trojans are outside the billion-year stability region, we numerically integrated the orbits of 36 real Trojan asteroids near the stability boundary. These Trojans are indicated in Figure 1 by either green or red dots. The current orbital elements for the

Trojans and the planets were kindly supplied to us by E. Bowell and L. Wasserman. The orbit of each Trojan was integrated for up to 4×10^9 years, or until it suffered a close encounter with one of the planets or was ejected from the solar system.

Of the 36 objects studied, 21 were unstable in less than 4 billion years. The objects we found to be unstable are represented by red dots in Figure 1, while those that were stable are shown in green. (Black dots represent Trojans that we did not integrate.) All the objects with current proper elements above the curve connecting the open triangles (those objects in our initial survey that were unstable after 10^9 years), were indeed unstable. In addition, all particles above Rabe's[8] analytical stability curve also were unstable. All the unstable Trojans followed similar behaviors, as illustrated in Figure 2.

The main result of the work presented in this *Letter* is that the Trojan swarms of Jupiter are slowly dispersing. Indeed, our results show that the Trojan swarms must be added to the growing list of dynamically unstable structures in the solar system.

One interesting side issue are the fates of the objects that escape the swarms. In order to address this, we integrated their orbits under the gravitational influence of the Sun and the 4 giant planets using the *RMVS* integration scheme[15]. Our integration scheme is based on the method of Wisdom & Holman[13], but it can handle close encounters between particles and planets. The trajectory of each escaped Trojan was followed for 2×10^7 years unless it either was ejected from the solar system, or impacted the Sun or a planet.

We found that, as first suggested by Rabe[8], escaped Trojans evolve onto orbits similar to those of Jupiter-family comets[16] (see Figure 3). It is pertinent to ask how many, if any, of the observed Jupiter-family comets are escaped Trojans, assuming that Trojans can become active like comets. A total of 178 known Trojans are shown in Figure 1. Of these, 21 (12%) are unstable over a 4 billion year time interval, assuming that the ones that we did not integrate are stable. This assumption is most likely valid since the remaining objects are well within our derived stability limit. The probability, p, that any given object will leave the swarm in a year is $0.12/4 \times 10^9 = 2.9 \times 10^{-11}\, yr^{-1}$. We estimate that there are approximately 2×10^6 Jupiter Trojan asteroids with radii greater than $1\, km$, the typical size of a comet (cf. Shoemaker et al.[1]). Thus, the leakage rate of bodies with diameters greater than $1\, km$ from the Trojan swarms due to dynamical erosion is $r = 6.2 \times 10^{-5}\, yr^{-1}$. Collisions may also play a role in knocking Trojans out of the swarm. In a recent paper, Marzari et al.[20] estimated r due to collisions is $10^{-3}\, yr^{-1}$, significantly higher then the rate from dynamical erosion.

The average number of escaped Trojans still resident in the solar system is $N_{tot} = r\, L$, where L is the mean time that an escaped object remains in the solar system and r is the sum of the rates due to collisions and dynamical erosion. From our integrations $L = 1.2 \times 10^6$ years, thus $N_{tot} = 1200$. However, these objects are spread throughout the solar system. Only $\sim 1\%$ have perihelia small enough to be easily discovered (defined as $q < 2.5\, AU$), which implies that no more than about 12 of the known Jupiter-family comets are escaped Trojans. The expected number that might be recognizable as comets is still less because the physical lifetimes of comets (the time over which they can produce a coma and tail) is much less than their dynamical lifetime, only about $5 - 20\%$ of comets are active[18,19]. If escaped Trojans behave like typical comets, then the total number of active Jupiter-family comets that are escaped Trojans is only ~ 1 out of the 150 known Jupiter-family comets.

Finally, it is interesting to inquire whether the comet D/Shoemaker-Levy 9, which impacted Jupiter in 1994, could have been an escaped Trojan asteroid of Jupiter. Figure 4 shows that this is quite unlikely because the orbital elements of recently escaped Trojans do not match the probable pre-capture orbital elements of D/Shoemaker-Levy 9.

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Figure Captions

Figure 1 — Isopleths of the dynamical lifetimes for Jupiter Trojans. Dots show the location of the known Trojan asteroids as a function of their proper elements. Owing to secular perturbations, the osculating orbital elements vary with time. However, it is possible to develop a set of *proper* orbital elements that remain approximately constant[12]. Trojan asteroids librate about points that are $\sim \pm 60^\circ$ from Jupiter. (The actual location of these fixed points are a function of the proper eccentricity[12].) If ϕ is defined as the instantaneous difference between the mean longitudes of the asteroid and Jupiter, then the libration amplitude, D, is the amplitude of oscillations in ϕ. The proper eccentricity, e_p, is defined as the amplitude of the oscillation in $e\cos(\tilde{\omega} - \tilde{\omega}_J)$, where e is the eccentricity of the asteroid and $\tilde{\omega}$ and $\tilde{\omega}_J$ are the longitudes of perihelia for the asteroid and Jupiter, respectively. The dashed orange curve shows the location of Rabe's[8] analytic stability limit. Notice that there are significant differences between Rabe's prediction and the observed location of the upper boundary of the swarms. Of particular interest here are the Trojans that are above Rabe's curve. Shoemaker, et al.[1] used these objects (except 1989 BQ and Achates which were not known at that time) to conclude that the true stability curve lies above Rabe's. The blue curves are contours of the dynamical lifetimes of particles in our first set of integrations, which were for $1\,Gyr$. The stable region shrinks quite dramatically over this time span. It is not possible to plot a contour at 10^9 years because the main integration was stopped at this point. In order to determine the limits for this isopleth, for each D we plot an open triangle showing the orbit with the smallest e_p that was unstable in $1\,Gyr$; the filled triangle represents with the largest e_p that was stable. The 10^9-year lifetime isopleth must lie between these two extremes. The color of a dot representing a known Trojan indicates whether the object is stable (green) or unstable (red) as determined by our second set of integrations; objects whose orbits we did not integrate are shown

in black.

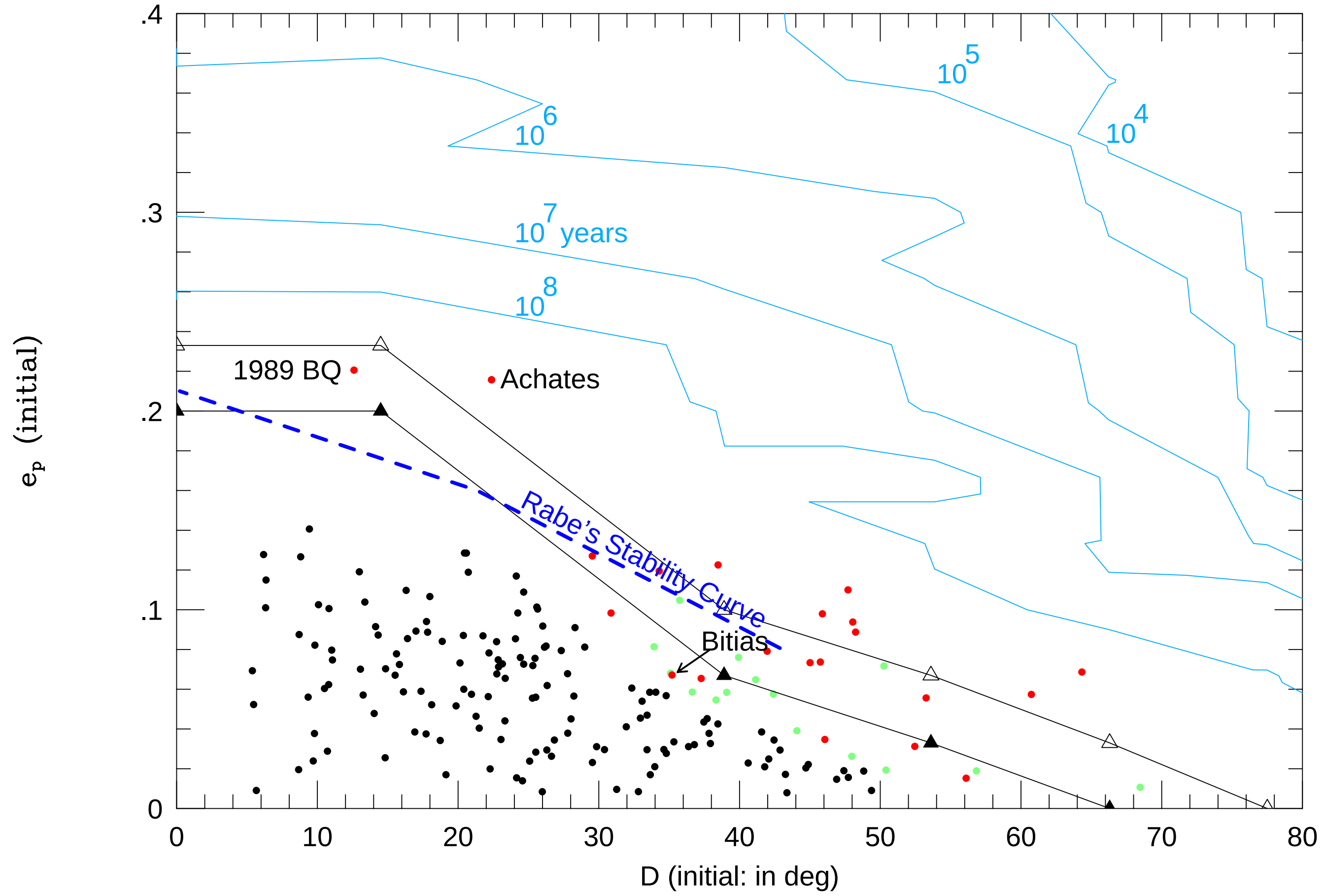
Figure 2 — Temporal variation of D and e_p of a typical Trojan on an unstable orbit, (5120) Bitias (labeled in Figure 1). The value of e_p is approximately constant while D increases as a function of time. Note that D remains approximately constant for over a billion years and starts to grow only near the end of the asteroid's life as a Trojan. This behavior is typical of the evolution of Trojans since all 21 unstable Trojans investigated behaved in a similar way. It is also apparently typical of chaotic orbits in other regions the solar system (see Levison & Duncan[14] for examples in the Kuiper Belt). It also explains why most of the unstable Trojans' proper eccentricities are within the same range of e_p found for stable Trojans. We infer that many of the unstable Trojans initially had smaller values of D and evolved onto their current orbits in the process of leaving the swarm. Additionally, many of the unstable Trojans may have been displaced in D from more stable orbits by collisions[1]. If indeed e_p remains approximately constant, it is difficult to understand the origin of the unstable objects (4835) 1989BQ and (5144) Achates, which have large e_p (see Figure 1). There are two possible explanations for these high eccentricity orbits: *i*) It is possible that these objects initially had very small values of D and we are currently observing them as they slowly evolve out of the swarms. This seems unlikely as there are no other known objects in the swarms with such large values of e_p. *ii*) These high-e_p orbits may be the result of collisions.

Figure 3 — Comparison between the orbital elements of the observed comets with periods less than 200 years and those calculated for escaped Trojans. a) The inclinations of the known periodic comets verse of their Tisserand parameters. The Tisserand parameter is usually defined as $T = a_J/a + 2\sqrt{(1-e^2)a/a_J}\ \cos(i)$ where a_J is Jupiter's semi-major axis and a, e, and i are the comet's semi-major axis, eccentricity and inclination, respectively. It is an approximation to the Jacobi

constant, which is an integral of the motion in the circular restricted three-body problem. It is also a measure of the relative velocity between a comet and Jupiter during close encounters, $v_{rel} \propto \sqrt{3-T}$. Jupiter-family comets are defined[16] as those with $T > 2$; they lie to the left of the dashed line in the figure. This limit on T implies that Jupiter-family comets have small encounter velocities with Jupiter. The shaded areas represent regions that are physically unattainable for cometary orbits. b) The inclination and Tisserand parameter of the escaped Trojans asteroids. Each object is plotted once when it first evolves onto an orbit with a perihelion distance less than $2.5\,AU$, where it is assumed that the object, if it is active, could be discovered by the usual methods of telescopic detection (see discussions by Duncan, Quinn, & Tremaine[17] and Levison & Duncan[18]). The discovery of comets with $q > 2.5AU$ is far from complete. This figure shows that the orbital element distribution of recently escaped Trojans is very similar to that of the known Jupiter-family comets.

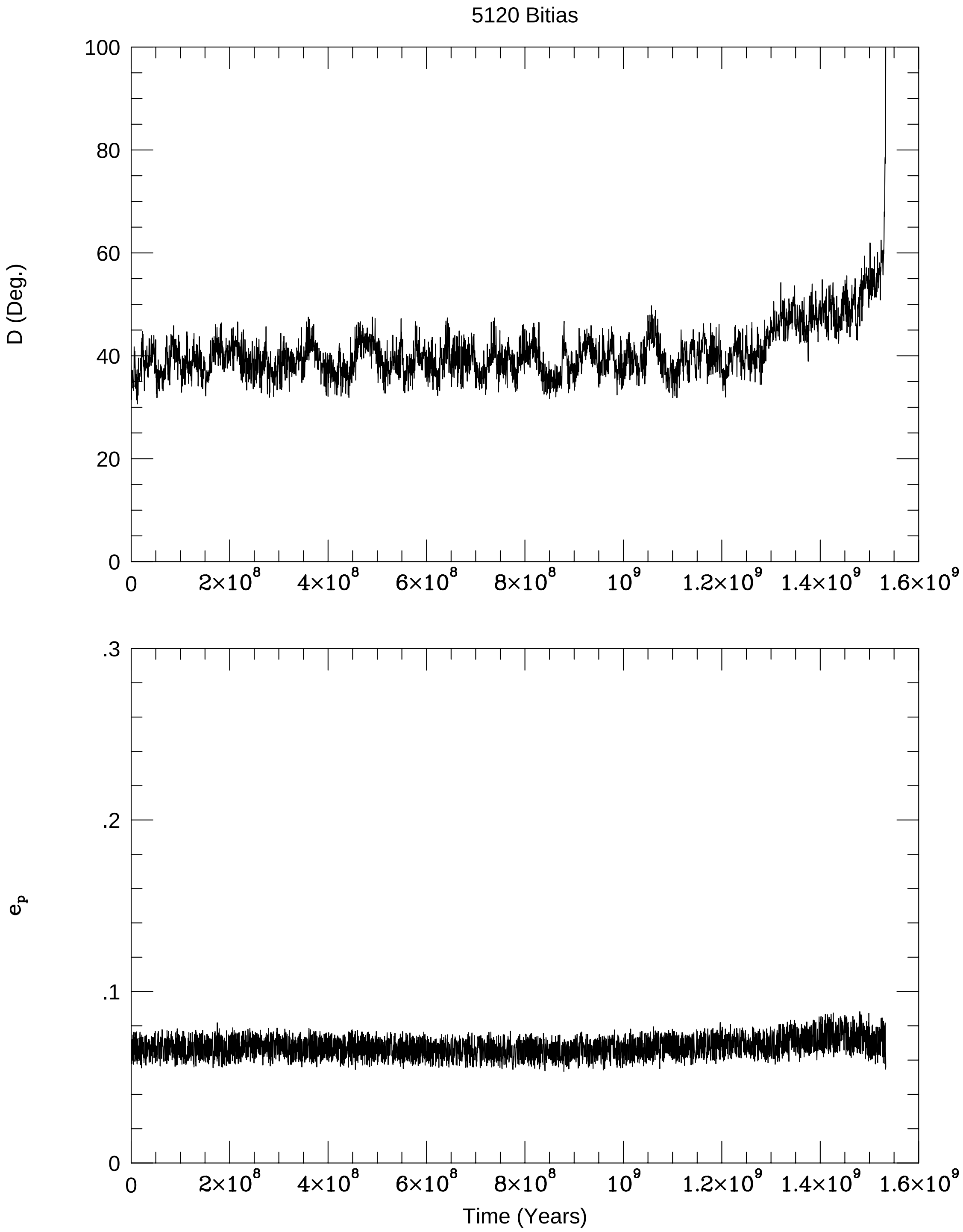
Figure 4 — A comparison between the orbital elements of escaped Trojans just after they first encounter Jupiter (large dots) and the possible precapture orbits of the comet D/Shoemaker-Levy 9 (small dots) as determined by Chodas and Yeomans[21]. The figure shows that there is very little if any overlap between the two distributions, indicating that D/Shoemaker-Levy 9 was not likely to be an escaped Trojan.

FIGURE 1



.4
.3
.2
.1
0
0
10
20
30
40
50
60
70
80
D (initial: in deg)
e_p (initial)
10^5
10^4
10^6
10^7 years
10^8
1989 BQ
Achates
Rabe's Stability Curve
Bitias

FIGURE 2



5120 Bitias
D (Deg.)
100
80
60
40
20
0
0
2×10^8
4×10^8
6×10^8
8×10^8
10^9
1.2×10^9
1.4×10^9
1.6×10^9
e_p
.3
.2
.1
0
Time (Years)

FIGURE 3

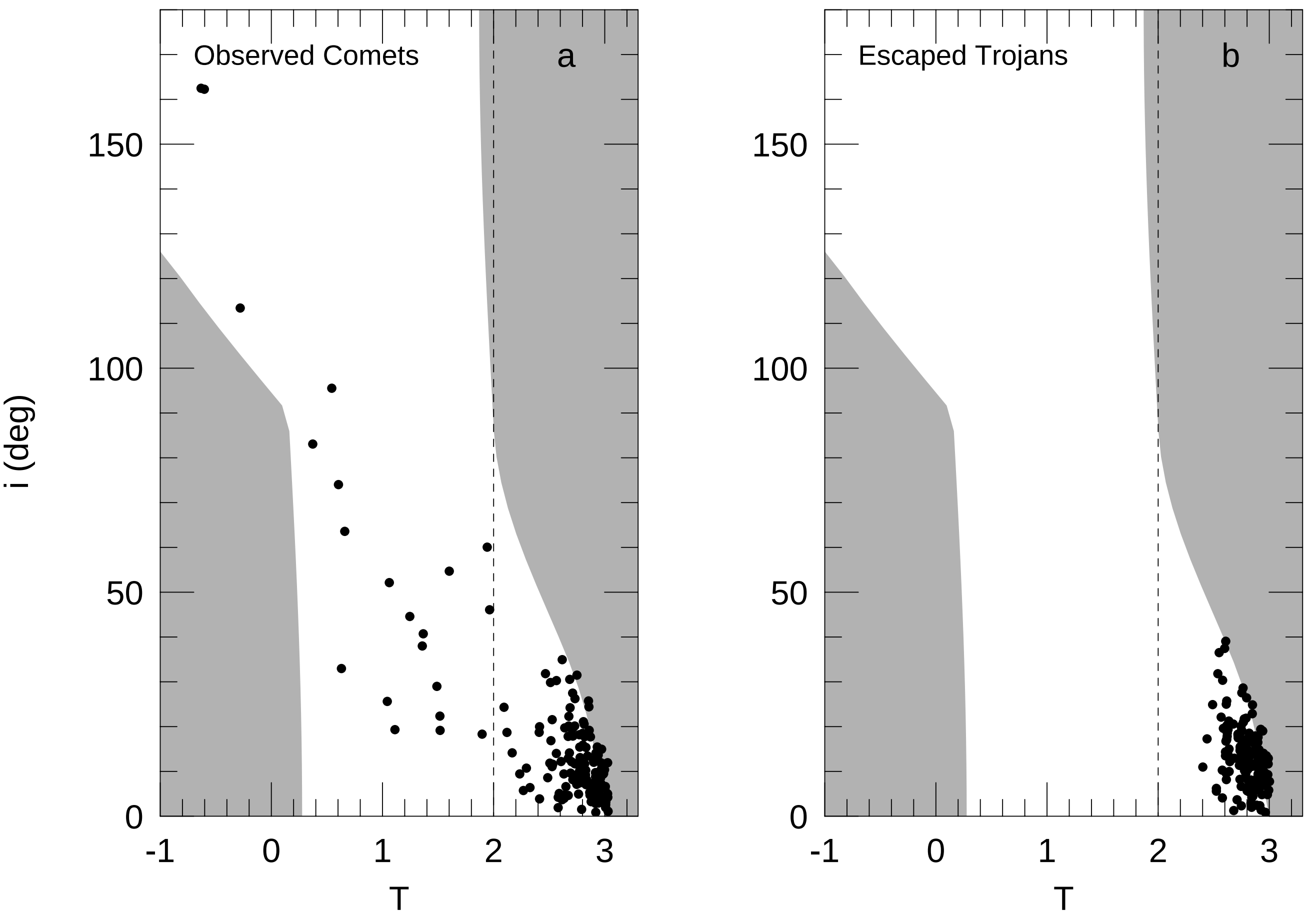


FIGURE 4