DYNAMICS OF THE KUIPER BELT

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Our current knowledge of the dynamical structure of the Kuiper Belt is reviewed here. Numerical results on long term orbital evolution and dynamical mechanisms underlying the transport of objects out of the Kuiper Belt are discussed. Scenarios about the origin of the highly non-uniform orbital distribution of Kuiper Belt objects are described, as well as the constraints these provide on the formation and long term dynamical evolution of the outer Solar system. Possible mechanisms include an early history of orbital migration of the outer planets, a mass loss phase in the outer Solar system and scattering by large planetesimals. The origin and dynamics of the scattered component of the Kuiper Belt is discussed. Inferences about the primordial mass distribution in the trans-Neptune region are reviewed. Outstanding questions about Kuiper Belt dynamics are listed.

I. INTRODUCTION

In the middle of this century, Edgeworth (1943) and Kuiper (1951) independently suggested that our planetary system is surrounded by a disk of material left over from the formation of planets. Both authors considered it unlikely that the proto-planetary disk was abruptly truncated at the orbit of Neptune. Each also suggested that the density in the Solar nebula was too small beyond Neptune for a major planet to have accreted, but that this region may be inhabited by a population of planetesimals. Edgeworth (1943) even suggested that bodies from this region might occasionally migrate inward and become visible as short-period comets. These ideas lay largely dormant until the 1980's, when dynamical simulations (Fernandez 1980; Duncan, Quinn, & Tremaine 1988; Quinn, Tremaine, & Duncan 1990) suggested that a disk of trans-Neptunian objects, now known as the *Kuiper belt*, was a much more likely source of the Jupiter-family short period comets than was the distant and isotropic Oort comet cloud.

With the discovery of its first member in 1992 by Luu and Jewitt (1993), the Kuiper belt was transformed from a theoretical construct to a *bona fide* component of the solar system. By now, on the order of 100 Kuiper Belt objects (KBOs) have been discovered - a sufficiently large number that permits first-order estimates about the mass and spatial distribution in the trans-Neptunian region. A comparison of the observed orbital properties of these objects with theoretical studies provides tantalizing clues to the formation and evolution of the outer Solar system. Other chapters in this book describe the observed physical and orbital properties of KBOs (Jewitt and Luu) and their collisional evolution (Farinella et al); here we focus on the dynamical structure of the trans-Neptunian region and the dynamical evolution of bodies in it. The outline of this chapter is as follows: in section II we review numerical results on long term dynamical stability of small bodies in the outer solar system; in section III we review our current understanding of the Kuiper Belt phase space structure and dynamics of orbital resonances and chaotic transport of KBOs; section IV provides a discussion of resonance sweeping and other mechanisms for the origin and properties of KBOs at Neptune's mean motion resonances; the origin and dynamics of the 'scattered disk' component of the Kuiper Belt is discussed in section V; current ideas about the primordial Kuiper Belt are described in section VI; we conclude in section VI with a summary of outstanding questions in Kuiper Belt dynamics.

II. LONG TERM ORBITAL STABILITY IN THE OUTER SOLAR SYSTEM

A. Trans-Neptune region

We first consider the orbital stability of test particles in the trans-Neptunian region and the implications for the resulting structure of the Kuiper belt several Gyrs after its presumed formation. Torbett (1989) performed direct numerical integration of test particles in this region including the perturbative effects of the four giant planets, although the latter were taken to be on fixed Keplerian orbits. He found evidence for chaotic motion with an inverse Lyapunov exponent (i.e. timescale for divergence of initially adjacent orbits) on the order of Myrs for moderately eccentric, moderately inclined orbits with perihelia between 30 and 45 AU (a "scattered disk"). Torbett & Smoluchowski (1990) extended this work and suggested that even particles with initial eccentricities as low as 0.02 are typically on chaotic trajectories if their semimajor axes are less than 45 AU. Except in a few cases, however, the authors were unable to follow the orbits long enough to establish whether or not most chaotic trajectories in this group led to encounters with Neptune. Holman & Wisdom (1993) and Levison & Duncan (1993) showed that indeed some objects in the the belt were dynamically unstable on timescales of Myr–Gyr, evolving onto Neptune-encountering orbits, thereby potentially providing a source of Jupiter–family comets at the present epoch.

Duncan, Levison & Budd (1995) performed integrations of thousands of particles for up to 4 Gyr in order to complete a dynamical survey of the trans-Neptunian region. The main results can be seen in Fig. 1 and include the following features:
i) For nearly circular, very low inclination particles there is a relatively stable band between 36 and 40 AU, with essentially complete stability beyond 42 AU. The lack of observed KBOs in the region between 36 and 42 AU suggests that some mechanism besides the dynamical effects of the planets in their current configuration must be responsible for the orbital element distribution in the Kuiper belt. This mechanism is almost certainly linked to the formation of the outer planets. Several possible mechanisms are described below.
ii) For higher eccentricities (but still very low inclinations) the region interior to 42 AU is largely unstable except for stable bands near mean-motion resonances with Neptune (e.g. the well-known 2:3 near 39.5 AU within which lies Pluto). The boundaries of the stable regions for each resonance have been computed independently by Morbidelli et al. (1995) and Malhotra (1996), and are in good agreement with the results shown in Fig. 1. The dynamics within the mean motion resonances are discussed in detail in section III.
iii) The dark vertical bands between 35-36 AU and 40-42 AU are particularly unstable regions in which the particles' eccentricities are driven to sufficiently high values that they encounter Neptune. These regions match very well the locations of the ν_7 and ν_8 secular resonances as computed analytically by Knezevic et al. (1991) – i.e., the test particles precess in these regions with frequencies very close to two of the characteristic secular frequencies of the planetary system.
iv) A comparison of the observed orbits of KBOs with the phase space structure (Figs. 1,2) shows that virtually all of the bodies interior to 42 AU are on moderately eccentric orbits and located in mean-motion resonances, whereas most of those beyond 42 AU appear to be in non-resonant orbits of somewhat lower eccentricities. In addition, there is one observed object, 1996 TL66, with semimajor axis ≈ 80 AU, beyond the range of Figs. 1,2. It is thought to be a member of a third class of KBOs representing a "scattered disk" (see section V). Attempts to understand the origin of these three broad classes of KBOs will occupy much of what follows in this chapter.

B. Test particle stability between Uranus and Neptune

Gladman & Duncan (1990) and Holman & Wisdom (1993) performed long term integrations, up to several hundred million years duration, of the evolution of test particles on initially circular orbits in-between the giant planets' orbits. The majority of the test particles were perturbed into a close approach to a planet on timescales of 0.01 – 100 Myr, suggesting that these regions should largely be clear of residual planetesimals. However, Holman (1997) has shown that there is a narrow region, 24 – 26 AU, lying between the orbits of Uranus and Neptune in which roughly 1% of minor bodies could survive on very low eccentricity and low inclination orbits for the age of the solar system. He estimated that a belt of mass totaling roughly $10^{-3}M_\oplus$ cannot be ruled out by current observational surveys. This niche is, however, extremely fragile. Brunini & Melita (1998) have shown that any one of several likely perturbations (e.g. mutual scattering, planetary migration, and Pluto-sized perturbers) would have largely eliminated such a primordial population. We note also that there are similarly stable (possibly even less 'fragile'), but apparently unpopulated dynamical niches in the outer asteroid belt (Duncan 1994) and the inner Kuiper belt (Duncan et al 1995).

C. Neptune Trojans

The only observational survey of which we are aware specifically designed to search for Trojans of planets other than Jupiter covered 20 square degrees of sky to limiting magnitude $V=22.7$ (Chen el al. 1997). Although 93 Jovian Trojans were found, no Trojans of Saturn, Uranus or Neptune were discovered. Although this survey represents the state-of-the-art, it lacks the sensitivity and areal coverage to reject the possibility that Neptune holds Trojan swarms similar in magnitude to those of Jupiter (Jewitt 1998, personal communication). Further searches clearly need to be done.

Several numerical studies of orbital stability in Neptune's Trojan regions have been published. Mikkola & Innanen (1992) studied the behavior of 11 test particles initially near the Neptune Trojan points for 2×10^6 years. Holman & Wisdom (1993) performed a 2×10^7 year integration of test particles initially in near-circular orbits near the Lagrange points of all the outer planets. And most recently, Weissman & Levison (1997) integrated the orbits of 70 test particles in the L4 Neptune Trojan zone for 4 Ga. In all these studies, some Neptune Trojan orbits were found to be stable. Weissman & Levison (1997) found that stable Neptune Trojans must have libration amplitudes $D \lesssim 60°$ and proper eccentricities $e_p \lesssim 0.05$. It is interesting to note that this range for D is similar to that Levison et al. (1997) found for the Jupiter Trojans, but the maximum stable e_p for the Neptune regions is a factor of three smaller than that of the Jupiter regions. Holman & Wisdom reported a curious asymmetric displacement of the L4 and L5 Trojan libration centers of Neptune whose cause remains unknown.

III. RESONANCE DYNAMICS AND CHAOTIC DIFFUSION

It is evident from the numerical analysis of test particle stability in the trans-Neptunian region that the timescales for orbital instability span several orders of magnitude and are very sensitive to orbital parameters. For example, the map of stability time (i.e., time to first encounter within a Hill sphere radius of Neptune) for initially circular orbits is very patchy, with short dynamical lifetimes interspersed amongst very stable regions (see Fig. 1). Most particles that have a close approach to Neptune are removed from the Kuiper Belt shortly thereafter by means of a quick succession of close approaches to the giant planets. However, a small fraction evolve into anomalously long-lived chaotic orbits beyond Neptune that do not have a second close approach to the planet on timescales comparable to the age of the Solar system (see section V). The nature and origin of the long-timescale instabilities (which are most relevant for understanding the origin of short period comets from the Kuiper Belt) is not well understood at present.

In general, we understand that the mean motion resonances of Neptune form a 'skeleton' of the phase space (Fig. 2), with the perturbations of the other giant planets, including secular resonances, forming a web of superstructure on that skeleton. The phase space in the neighborhood of Neptune's 3:2 mean motion resonance is the best studied, following the discovery of Pluto and its myriad of resonances (cf., Malhotra & Williams, 1997). Fig. 3 shows the dynamical features that have been identified at the 3:2 resonance, in the (a, e) plane. The boundary of this resonance, determined by means of a semi-numerical analysis of the averaged perturbation potential of Neptune, is shown (dark solid lines), as well as the loci of the apsidal ν_8 and nodal ν_{18} secular resonances and the Kozai resonance in this neighborhood (Morbidelli, 1997). The stable libration zone, determined from an inspection of many surfaces of section of the planar restricted 3-body model of the Sun-Neptune-Plutino (Malhotra, 1996), is indicated by the blue shaded region. The stable resonance libration boundary is significantly different from the formal perturbation theory result because averaging is not a good approximation in the vicinity of the resonance separatrix: the separatrix broadens into a chaotic zone owing to the interaction with neighboring mean motion resonances. The width of the chaotic separatrix increases with eccentricity, eventually merging with the chaotic separatrices of neighboring mean motion resonances. The ν_8 secular resonance is mostly embedded in the chaotic zone, while the ν_{18} occurs at large libration amplitudes close to the chaotic separatrix; the Kozai resonance occurs at large libration amplitude for low eccentricity orbits, and at smaller libration amplitude for eccentricities near 0.2–0.25.

Fig. 4 shows a "map" of the diffusion speed of test particles determined by numerical integrations of up to 4 billion years by Morbidelli (1997). It is clear that instability timescales in the vicinity of the 3:2 resonance range from less than a million years to longer than the age of the Solar system. In general, within the resonance, higher eccentricity orbits are less stable than lower eccentricities; the stability times are longest deep in the resonance and shorter near the boundaries. We note that the uncolored regions exterior and adjacent to the colored zones at eccentricities below ~ 0.15 are stable for the age of the Solar system, but those above ~ 0.15 are actually chaotic on timescales shorter than the shortest indicated in the colored zones.

The short stability timescales in the most unstable zones are due to dynamical chaos generated by the interaction with neighboring mean motion resonances; this can be directly visualized in surfaces of section of the planar restricted three body model (Malhotra, 1996). Test particles in these zones suffer large chaotic changes in semimajor axis and eccentricity on short timescales, $\mathcal{O}(10^{5-7})$ yr, and are not protected from close encounters with Neptune. In a small zone near semimajor axis 39.5 AU, initially circular low-inclination orbits are excited to high eccentricity *and* high inclination on timescales of $\mathcal{O}(10^7)$ yr (Holman & Wisdom (1993), Levison & Stern (1997)); this instability is possibly due to overlapping secondary resonances (Morbidelli, 1997). The finite diffusion timescale, comparable to but shorter than the age of the Solar system, in a large area (green zone) inside the resonance is not understood at all; possibly higher order secondary resonances are the underlying cause. The numerical evolution of test particles in this region follows initially a slow diffusion in semimajor axis with nearly constant mean eccentricity and inclination, until the orbit eventually reaches a strongly chaotic zone (Morbidelli, 1997). The diffusion timescales in this region are comparable to or only slightly less than the age of the Solar system, so that it is likely an active source region for short period comets via purely dynamical instabilities.

The dynamical structure in the vicinity of other mean motion resonances has not been obtained in as much detail as that of the 3:2. We expect differences in details – different profile of the libration zone, differing secular resonance effects, etc. – but generally similar qualitative features. We also note that since the orbital evolution obtained in non-dissipative models of Kuiper Belt dynamics is time-reversible, the transport of particles from strongly chaotic regions to weakly chaotic regions is also allowed. In the most general terms, this is the likely explanation for the putative Scattered Disk (section V).

IV. RESONANT KUIPER BELT OBJECTS

The origin of the great abundance of resonant KBOs and, equally importantly, their high orbital eccentricities, is an interesting question whose understanding may lead to significant advances in our understanding of the early history of the Solar system. In this section, we discuss current ideas pertinent to this class of KBOs.

A. Planet migration and resonance sweeping

An outward orbital migration of Neptune in early solar system history provides an efficient mechanism for sweeping up large numbers of trans-Neptunian objects into Neptune's mean motion resonances. We describe this scenario in some detail here; its importance stems from the linkage it provides between the detailed orbital distribution in the Kuiper Belt and the early orbital migration history of the outer planets. This theory, which was originally proposed for the origin of Pluto's orbit (Malhotra, 1993), supposes that Pluto formed in a common low-eccentricity, low-inclination orbit beyond the (initial) orbit of Neptune. It was captured into the 3:2 resonance with Neptune and had its eccentricity excited to its current Neptune-crossing value as Neptune's orbit expanded outward due to angular momentum exchange with residual planetesimals. The theory predicts that resonance capture and eccentricity excitation would be a common fate of a large fraction of trans-Neptunian objects (Malhotra, 1995).

The process of orbital migration of Neptune (and, by self-consistency, migration of the other giant planets) invoked in this theory is as follows. Consider the late stages of planet formation when the outer Solar system had reached a configuration close to its present state, namely four giant planets in well separated near-circular, co-planar orbits, the nebular gas had already dispersed, the planets had accreted most of their mass, but there remained a residual population of icy planetesimals and possibly larger planetoids. The subsequent evolution consisted of the gravitational scattering and accretion of these small bodies. Circumstantial evidence for this exists in the obliquities of the planets (Lissauer & Safronov, 1991; Dones & Tremaine, 1993). Much of the Oort Cloud – the putative source of long period comets – would have formed during this stage by the scattering of planetesimals to wide orbits by the giant planets and the subsequent action of galactic tidal perturbations and perturbations from passing stars and giant molecular clouds (e.g. Fernandez, 1985; Duncan et al, 1987). During this era, the back reaction of planetesimal scattering on the planets could have caused significant changes in their orbital energy and angular momentum. Overall, there was a net loss of energy and angular momentum from the planetary orbits, but the loss was not extracted uniformly from the four giant planets. Jupiter, by far the most massive of the planets, likely provided all of the lost energy and angular momentum, and more; Saturn, Uranus and Neptune actually gained orbital energy and angular momentum and their orbits expanded, while Jupiter's orbit decayed sufficiently to balance the books. This was first pointed out by Fernandez & Ip (1984) who noticed it in numerical simulations of the late stages of accretion of planetesimals by the proto-giant planets.

Migration of the Jovian planets

The reasons for the rather non-intuitive result can be understood from the following heuristic picture of the clearing of a planetesimal swarm from the vicinity of Neptune. Suppose that the mean specific angular momentum of the swarm is initially equal to that of Neptune. A small fraction of planetesimals is accreted as a result of physical collisions. Of the remaining, there are approximately equal numbers of inward and outward scatterings. To first order, these cause no net change in Neptune's orbit. However, the subsequent fate of the inward and outward scattered planetesimals is not symmetrical. Most of the inwardly scattered objects enter the zones of influence of Uranus, Saturn and Jupiter. Of those objects scattered outward, some are eventually lifted into wide, Oort Cloud orbits while most return to be rescattered; a fraction of the latter is again rescattered inwards where the inner Jovian planets, particularly Jupiter, control the dynamics. The massive Jupiter is very effective in causing a systematic loss of planetesimal mass by ejection into Solar system escape orbits. As Jupiter preferentially removes the inward scattered planetesimals from Neptune's influence, the planetesimal population encountering Neptune at later times is increasingly biased towards objects with specific angular momentum and energy larger than Neptune's. Encounters with this planetesimal population produce effectively a negative drag on Neptune which increases its orbital radius. Uranus and Saturn, also being much less massive than and exterior to Jupiter, experience a similar orbital migration, but smaller in magnitude than Neptune. Thus Jupiter is, in effect, the source of the angular momentum and energy needed for the orbital migration of the outer giant planets, as well as for the planetesimal mass loss. However, owing to its large mass, its orbital radius decreases by only a small amount.

The magnitude and timescale of the radial migration of the Jovian planets due to their interactions with residual planetesimals is difficult to determine without a full-scale N-body model. The work of Fernandez & Ip (1984) is suggestive but not conclusive due to several limitations of their numerical model which produced highly stochastic results: they modeled a small number of planetesimals, ~ 2000, and the masses of individual planetesimals were in the rather exaggerated range of 0.02–0.3 $M_\oplus$; and, perhaps most significantly, they neglected long range gravitational forces. Current studies attempt to overcome these limitations by using the faster computers now available and more sophisticated integration algorithms (Hahn & Malhotra, 1998). Still, fully self-consistent high fidelity models remain a distant goal at this time. Remarkably, an estimate for the magnitude and timescale of

Neptune's outward migration is possible from an analysis of the orbital evolution of KBOs captured in Neptune's mean motion resonances.

Resonance sweeping

Capture into resonance is a delicate process, difficult to analyze mathematically. Under certain simplifying assumptions, Malhotra (1993, 1995) showed that resonance capture is very efficient for adiabatic orbital evolution of KBOs whose initial orbital eccentricities were smaller than ~ 0.05. Resonance capture leads to an excitation of orbital eccentricity whose magnitude is related to the magnitude of orbital migration:

$$\Delta e^2 \simeq \frac{k}{j+k} \ln \frac{a_f}{a_i} = \frac{k}{j+k} \ln \frac{a_N}{a_{N,i}}, \tag{1}$$

where a_i and a_f are the initial and current semimajor axes of a KBO, a_N is Neptune's current semimajor axis and $a_{N,i}$ is its value in the past at the time of resonance capture; j and k are positive integers defining a $j : j+k$ mean motion resonance. From this equation and the observed eccentricities of Pluto and the Plutinos (see Jewitt & Luu, this vol.), it follows that Neptune's orbit has expanded by $\sim$9 AU.

Numerical simulations of resonance sweeping of the Kuiper Belt have been carried out assuming adiabatic giant planet migration of specified magnitude and timescale. The orbital distribution of Kuiper Belt objects obtained in one such simulation is shown in Fig. 5. The main conclusions from such simulations are that (i) few KBOs remain in circular orbits of semimajor axis $a \lesssim 39$ AU which marks the location of the 3:2 Neptune resonance; (ii) most KBOs in the region up to $a = 50$AU are locked in mean motion resonances; the 3:2 and 2:1 are the dominant resonances, but the 4:3 and 5:3 also capture noticeable numbers of KBOs; (iii) there is a significant paucity of low-eccentricity orbits in the 3:2 resonance; (iv) the maximum eccentricities in the resonances are in excess of Neptune-crossing values; (v) inclination excitation is not as efficient as eccentricity excitation: only a small fraction of resonant KBOs acquire inclinations in excess of 10°. Not shown in Fig. 5 are other dynamical features such as the resonance libration amplitude and the argument-of-perihelion behavior (libration, as for Pluto, or circulation), which are also reflective of the planet migration/resonance sweeping process.

More detailed discussion of numerical results on resonance sweeping is given in Malhotra (1995,1997,1998a,1999) and Holman (1995). Two additional points are worthy of note here. One is that, owing to the longer dynamical timescales associated with vertical resonances, the magnitude of inclination excitation of KBOs is sensitive to both the magnitude and the timescale of planetary migration; it is estimated that a timescale on the order of $(1-3) \times 10^7$ yr would account for Pluto's inclination (Malhotra, 1998a). The second is that the total mass of residual planetesimals required for a $\sim$ 9 AU migration of Neptune is estimated to be $\sim 50 M_\oplus$ (Malhotra, 1997); this estimate is supported by recent numerical simulations of planet migration (Hahn & Malhotra, 1998).

An outstanding issue is the apparent paucity of KBOs at Neptune's 2:1 mean motion resonance in the observed sample (Fig. 2)*, whereas the planet migration/resonance sweeping theory predicts comparable populations in the 3:2 and 2:1 resonances (Fig. 5). Possible explanations are: (i) observational incompleteness (cf., Gladman et al, 1998), or (ii) significant leakage out of the 2:1 resonance on billion year timescales by means of weak instabilities or perturbations by larger members of the Kuiper Belt (Malhotra, 1999), or (iii) the planet migration/resonance sweeping did not occur as postulated. However, the success of the resonance sweeping mechanism in explaining the orbital eccentricity distribution of Plutinos – and the difficulty of explaining it by other means – argues strongly in its favor. Further work is needed to refine the relationship between the orbital element distributions and the detailed characteristics of the planet migration process, including the overall efficiency of resonance capture and retention.

If such planet migration and resonance sweeping occurred, then it follows that the KBOs presently resident in Neptune's mean motion resonances formed closer to the Sun than their current semimajor axes would suggest. If there were a significant compositional gradient in the primordial trans-Neptunian planetesimal disk, it may be preserved in a rather subtle manner in the present orbital distribution. Because each resonant KBO retains memory of its initial orbital radius in its final orbital eccentricity (Eq. 1), there would exist a compositional gradient with orbital eccentricity within each resonance; non-resonant KBOs in near-circular low-inclination orbits between 30 AU and 50 AU most likely formed at their present locations and would reflect the primordial conditions at those locations. However, if there were significant orbital mixing by processes other than resonance sweeping, these systematics would be diluted or erased.

B. Secular resonance sweeping

The combined, orbit-averaged perturbations of the planets on each other cause a slow precession of the direction of perihelion and of the pole of the orbit plane of each planet. Of particular importance to the long term dynamics of the Kuiper Belt are the perihelion and orbit pole precession of Neptune, both of which have periods of about 2 Myr in the present planetary configuration. The perihelion direction and orbit

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* In December 1998, while this article was in the review process, the Minor Planet Center reported new observations yielding revised orbits for 1997 SZ10 and 1996 TR66, identifying these as the first two KBOs in the 2:1 resonance with Neptune (Marsden, 1998).

pole orientation of the orbits of Kuiper Belt objects also precess slowly, at rates that depend upon their orbital parameters. For certain ranges of orbital parameters, the perihelion precession rate matches that of Neptune; this condition is termed the ν_8 *secular resonance*. Similarly, the 1:1 commensurability of the rate of precession of the orbit pole with that of Neptune's orbit pole is called ν_{18} *secular resonance*. (See Malhotra (1998b) for an analytical model of secular resonance.) The ν_8 and ν_{18} secular resonances occur at several regions in the Kuiper Belt where they cause strong perturbations of the orbital eccentricity and orbital inclination, respectively (cf. Fig. 1,3).

The secular effects are sensitive to the mass distribution in the planetary system (see Ward 1981, and references therein). Levison et al. (1999) have noted that a primordial massive trans-Neptunian disk would have significantly altered the locations of the ν_8 and ν_{18} secular resonances. From a suite of numerical simulations, they estimate that a $\sim 10 M_\oplus$ primordial disk between 30 AU and 50 AU would have the ν_8 secular resonance near $a \lesssim 36$ AU, and that it would have moved outward to its current location near 42 AU as the disk mass declined. Such sweeping by the ν_8 secular resonance excites the orbital eccentricities of KBOs sufficiently to cause them to encounter Neptune and be removed from the Kuiper Belt. Only objects fortuitously trapped in Neptune's mean motion resonances remain stable. The simulations also find that the ν_{18} secular resonance sweeps *inward* from well beyond its current location as the Kuiper Belt mass declines, thereby moderately increasing the inclinations (up to $\sim$ 15 degrees) of KBOs beyond 42 AU. However, this mechanism does not produce orbital eccentricities in Neptune's 2:3 mean motion resonance as large as those observed. Furthermore, damping of the eccentricity and inclination by density waves is to be expected in a massive primordial disk (Ward & Hahn, 1998), but has not yet been included in the simulations. We conclude that the sweeping of secular resonances has probably played some role in the excitation of the Kuiper belt, but its quantitative effects remain to be determined.

C. Stirring by Large Neptune-scattered Planetesimals

A third mechanism to explain the observed orbital properties of KBOs is to invoke the orbital excitation produced by close encounters with large Neptune-scattered planetesimals on their way out of the solar system or to the Oort cloud. The observed excitation in the Kuiper belt requires the prior existence of planetesimals with masses ~ 1 Earth mass according to Morbidelli and Valsecchi (1997). There is circumstantial evidence (e.g. the obliquity of Uranus' spin axis) that a population of objects this massive might have formed in the region between Uranus and Neptune and are now gone (Safronov 1966, Stern 1991). Many of these objects must have spent some time orbiting through some parts of the Kuiper belt. A much more massive initial belt might then have been sculpted to its presently observed structure because of the injection of most of the small bodies into dynamically unstable regions in the inner belt and the enhanced role of mutually catastrophic collisions among small planetesimals in the outer belt. In this picture, then, the observed KBOs interior to ~ 42 AU are the lucky ones ending up in the small fraction of phase space ($\sim$ few percent –see Figure 2) protected from close encounters with Neptune by mean-motion resonances. This mechanism is similar to one involving large Jupiter-scattered planetesimals proposed earlier to explain the excitation of the asteroid belt (Ip 1987; Wetherill 1989).

Petit et al. (1998) have combined 3-body integrations and semi-analytic estimates of scattering to model the effects of large planetesimals in the Kuiper belt. They argue that the best reconstruction of the observed dynamical excitation of the Kuiper belt requires the earlier existence of two large bodies. The first is a body of half an Earth mass on an orbit of large eccentricity with a dynamical lifetime of several times 10^8 yr. The second is a body of 1 Earth mass, which evolves for ~ 25 Myr on a low eccentricity orbit spanning the 30-40 AU range.

An attractive feature of this mechanism is that it yields an overall mass depletion in the inner Kuiper Belt and accounts for the fact that the outer Kuiper belt ($a > 42$ AU) is moderately excited. It does not, however, appear to explain the lack of low-eccentricity objects in Neptune's 2:3. In addition, the models performed to date require a specific set of very large objects, for which there is no direct evidence, to be at the right place for the right length of time. The presumed eventual removal of these objects by means of a final close encounter with Neptune would perturb Neptune's orbit significantly, and also jeopardize the stability of resonant KBOs; this is in conflict with the observed evidence.

V. THE SCATTERED DISK

As noted previously, the current renaissance in Kuiper belt research was prompted by the suggestion that the Jupiter-family comets originated there (Fernández 1980; Duncan, Quinn, & Tremaine 1988). Thus, as part of the research intended to understand the origin of these comets, a significant amount of effort has gone into understanding the dynamical behavior of objects that are on orbits that can encounter Neptune (Duncan, Quinn, & Tremaine 1988; Quinn, Tremaine, & Duncan 1990; Levison & Duncan 1997). It is somewhat ironic, therefore, that these studies have led to the realization that a structure known as the *Scattered Comet Disk*, rather than the Kuiper belt, could be the dominant source of the Jupiter-family comets.

For our purposes, scattered disk objects are distinct from Kuiper belt objects in that they evolved out of their primordial orbits beyond Uranus early in the history of the solar system. These objects were then dynamically scattered by Neptune into orbits with perihelion distances near Neptune, but semi-major axes in the Kuiper belt (Duncan & Levison 1997). Finally, some process, usually interactions with Neptune's mean motion resonances, raised their perihelion distances thereby effectively storing the objects for the age of the solar system. Scattered disk objects (hereafter SDOs) occupy the same physical space as KBOs, but can be distinguished from KBOs by their orbital elements. In particular, as we describe in more detail below, SDOs tend to be on much more eccentric orbits than KBOs.

Of the 60 or so Kuiper belt objects thus far cataloged, only one, 1996 TL_{66}, is an obvious SDO. 1996 TL_{66} was discovered in October, 1996 by Jane Luu and colleagues (Luu et al. 1997) and is estimated to have a semi-major axis of 85 AU, a perihelion of 35 AU, an eccentricity of 0.59, and an inclination of 24°. Such a high eccentricity orbit most likely resulted from gravitational scattering by a giant planet, in this case Neptune.

The idea of the existence of a scattered comet disk dates to Fernández & Ip (1989). Their numerical simulations indicated that some objects on eccentric orbits with perihelia inside the orbit of Neptune could remain on such orbits for billions of years and hence might be present today. However, their simulations were based on an algorithm which incorporates only the effects of close gravitational encounters (Öpik, 1951), and hence severely overestimates the dynamical lifetimes of bodies such as those in their putative disk (Dones et al, 1998). As a result, the dynamics of their scattered disk bears little resemblance to the structure found in more recent direct integrations to be discussed below.

The only investigation of the scattered disk which uses modern direct numerical integrations is the one by Duncan & Levison (1997, hereafter DL97). DL97 was a followup to Levison & Duncan (1997), which was an investigation of the behavior of 2200 small, massless objects that initially were encountering Neptune. The latter's focus was to model the evolution of these objects down to Jupiter-family comets and followed the system for only 1 billion years. DL97 extended these integrations to 4 billion years. Most objects that encounter Neptune have short dynamical lifetimes. Usually, they are either 1) ejected from the solar system, 2) hit the Sun or a planet, or 3) are placed in the Oort cloud, in less than $\sim 5 \times 10^7$ years. It was found, however, that 1% of the particles remained in orbits beyond Neptune after 4 billion years. So, if at early times there was a significant amount of material from the region between Uranus and Neptune or the inner Kuiper belt that evolved onto Neptune-crossing orbits, then there could be a significant amount of this material remaining today. What is meant by 'significant' is the main question when it comes to the current importance of the scattered comet disk.

DL97 found that some of the long-lived objects were scattered to very long-period orbits where encounters with Neptune became infrequent. However, at any given time, the majority of them were interior to 100 AU. Their longevity is due in large part to their being temporarily trapped in or near mean-motion resonances with Neptune. The 'stickiness' of the mean motion resonances, which was mentioned by Holman & Wisdom (1993), leads to an overall distribution of semimajor axes for the particles that is peaked near the locations of many of the mean motion resonances with Neptune. Occasionally, the longevity is enhanced by the presence of the Kozai resonance. In all long-lived cases, particles had their perihelion distances increased so that close encounters with Neptune no longer occurred. Frequently, these increases in perihelion distance were associated with trapping in a mean motion resonance, although in many cases it has not yet been possible to identify the exact process that was involved. On occasion, the perihelion distance can become large, but 81% of scattered disk objects have perihelia between 32 and 36 AU.

Fig. 6 shows the dynamical behavior of a typical particle. This object initially underwent a random walk in semi-major axis due to encounters with Neptune. At about 7×10^7 years it was temporarily trapped in Neptune's 3:13 mean motion resonance for about 5×10^7 years. It then performed a random walk in semi-major axis until about 3×10^8 years, when it was trapped in the 4:7 mean motion resonance, where it remained for 3.4×10^9 years. Notice the increase in the perihelion distance near the time of capture. While trapped in this resonance, the particle's eccentricity became as small as 0.04. After leaving the 4:7, it was trapped temporarily in Neptune's 3:5 mean motion resonance for $\sim 5 \times 10^8$ yr and then went through a random walk in semi-major axis for the remainder of the simulation.

DL97 estimated an upper limit on the number of possible SDOs by assuming that they are the sole source of the Jupiter-family comets. DL97 computed the simulated distribution of comets throughout the solar system at the current epoch (averaged over the last billion years for better statistical accuracy). They found that the ratio of scattered disk objects to visible Jupiter-family comets* is 1.3×10^6. Since there are currently estimated to be 500 visible JFCs (Levison & Duncan 1997), there are $\sim 6 \times 10^8$ comets in the scattered disk if it is the sole source of the JFCs. It is quite possible that the scattered disk could contain this much material. Figure 7 shows the spatial distribution for this model.

* We define a 'visible' Jupiter-family comet as one with a perihelion distance less than 2.5 AU.

To review the above findings, $\sim 1\%$ of the objects in the scattered disk remain after 4 billion years, and that 6×10^8 comets are currently required to supply all of the Jupiter-family comets. Thus, a scattered comet disk requires an initial population of only 6×10^{10} comets (or $\sim 0.4 M_\oplus$, Weissman 1990) on Neptune-encountering orbits. Since planet formation is unlikely to have been 100% efficient, the original disk could have resulted from the scattering of even a small fraction of the tens of Earth masses of cometary material that must have populated the outer solar system in order to have formed Uranus and Neptune.

VI. THE PRIMORDIAL KUIPER BELT

As with many scientific endeavors, the discovery of new information tends to raise more questions than it answers. Such is the case with the Kuiper belt. Even the original argument that suggested the Kuiper belt is in doubt. Edgeworth's (1949) and Kuiper's (1951) speculations were based on the idea that it seemed unlikely that the disk of planetesimals that formed the planets would have abruptly ended at the current location of the outermost known planet, Neptune. An extrapolation into the Kuiper belt of the current surface density of non-volatile material in the outer planetary region implies that there should be about $30 M_\oplus$ of material there. However, the recent KBO observations indicate only a few $\times \sim 0.1 M_\oplus$ between 30 and $50\,AU$. Were Kuiper and Edgeworth wrong? Is there a sharp outer edge to the planetary system? One line of theoretical arguments suggests that the answer may be no to both questions; we discuss these below, but we note at the end some contrary arguments as well.

Over the last few years, several points have been made supporting the idea of a massive primordial Kuiper belt; see Farinella et al. (this volume). Stern (1995) and Davis & Farinella (1997) have argued that the inner part of the Kuiper belt ($\lesssim 50\,AU$) is currently eroding away due to collisions, and therefore must have been more massive in the past. Stern (1995) also argued that the current surface density in the Kuiper Belt is too low to grow bodies larger than about $30\,km$ in radius by means of two-body collisional accretion over the age of the Solar system. The observed $\sim$ 100 km size KBOs could have grown in a more massive Kuiper belt, of at least several Earth masses, if the mean eccentricities of the accreting objects were small, on the order of a few times 10^{-3} or perhaps as large as 10^{-2} (Stern 1996; Stern & Colwell 1997a; Kenyon & Luu 1998). Models of the collisional evolution of a massive Kuiper belt suggest that 90% of the mass inside of $\sim 50\,AU$ could have been been lost due to collisions over the age of the solar system (Stern & Colwell 1997b; Davis & Farinella 1998). Thus, it is possible for a massive primordial Kuiper belt to grind itself down to the levels that we see today.

With these arguments, it is possible to build a strawman model of the Kuiper belt, which is depicted in Fig. 8. Following Stern (1996), there are three distinct zones in the Kuiper belt. Region A is a zone where the gravitational perturbations of the outer planets have played an important role, tending to pump up the eccentricities of objects. About half of the objects in this region could have been dynamically removed from the Kuiper belt (Duncan et al. 1995). The remaining objects have eccentricities that are large enough that accretion has ceased and erosion is dominant. Thus, we expect that a significant fraction of the mass in region A has been removed by collisions. The dotted curve shows an estimate of the initial surface density of solids, extrapolated from that of the outer planets, and the solid curve (marked DLB95) shows an estimate of the current Kuiper belt surface density from Duncan et al. (1995, reproduced from their Fig. 7). Region B in Fig. 8 is a zone where collisions are important but the perturbations of the planets are not. The orbital eccentricities of objects in this region will most likely remain small enough that collisions lead to accretion of bodies, rather than erosion. Large objects could have formed here (see Stern & Colwell 1997a, hereafter SC97a), but the surface density may not have changed very much in this region over the age of the solar system. Observational constraints based on COBE/DIRBE data put the transition between Regions A and B beyond $70\,AU$ (Teplitz et al. 1999). The outer boundary of Region B is very uncertain, but will likely be beyond $100\,AU$ (Stern, personal communication). Region C is a zone where collision rates are low enough that the surface density of the Kuiper belt and the size distribution of objects in it have remained virtually unchanged over the history of the solar system.

If the above model is correct, then the Kuiper belt that we have so far observed may be a low density region that lies inside a much more massive outer disk. In other words, we may now be seeing a 'Kuiper Gap' (Stern & Colwell 1997b).

The idea of an increase in the surface density of the Kuiper belt beyond $50\,AU$ may explain a puzzling feature of the dynamics of the planets: Neptune's small eccentricity. Ward & Hahn (1998, hereafter WH98) have shown that if the Kuiper belt smoothly extends to a couple of hundred AU and if the eccentricities in the disk are smaller than ~ 0.05 then Neptune can excite apsidal waves in the disk that will carry away angular momentum from Neptune's orbit and damp that planet's eccentricity. They estimate an upper limit of $\sim 2M_{\oplus}$ on the mass of material between 50 and $75\,AU$. This upper limit is marked WH98 in Fig. 8.

We note that WH98 estimate more mass than a simple extension of the Kuiper belt's *observed* surface density would imply (i.e. we are seeing a Kuiper gap). However, WH98's upper limit on the surface density is much less than that required by the accretion models to grow the observed KBOs. There are three possible resolutions to this problem. *i*) It could be a matter of timing. As SC97a describe, the KBOs must have been formed before Neptune in order for their eccentricities to be small enough for accretion. Conversely, WH98's upper limit only applies after Uranus and Neptune have cleared away any local objects that could excite their eccentricities. Perhaps the surface density of the Kuiper belt decreased significantly between these two times. For example, it could have decreased due to Neptune and Uranus injecting massive objects into the 50-100 AU region which stirred up the disk and greatly increased the collisional erosion (Morbidelli & Valsecchi 1997). *ii*) The uncertainties in the models could be larger than the discrepancy between them. SC97a make assumptions about the velocity evolution of their objects and the physics of the collisions. WH98 make assumptions about the shape of the surface density distribution and eccentricities. Perhaps when more details models are constructed, the discrepancy will vanish. *iii*) There really was an edge to the disk from which the planets formed at about 50 AU. This would explain why no classical KBOs have been discovered beyond this point. Although the lack of discoveries may be due to observational incompleteness (Gladman et al. 1998), Monte Carlo simulations of the detection statistics of observational surveys are consistent with a Kuiper Belt edge at 50 AU (Jewitt et al., 1998). Clearly more work needs to be done on this topic.

VII. CONCLUDING REMARKS

Studies of Kuiper Belt dynamics offer the exciting prospect of deriving constraints on dynamical processes in the late stages of planet formation which have hitherto been considered beyond observational constraint. Many questions have been raised by the intercomparisons between observations of the Kuiper Belt and theoretical studies of its dynamics; these outstanding issues are listed below.

1. It is likely that the Kuiper Belt defines an outer boundary condition for the primordial planetesimal disk, and by extension, for the primordial Solar Nebula. What new constraints does it provide on the Solar Nebula, its spatial extent and surface density, and on the timing and manner of formation of the outer planets Uranus and Neptune? How does the Kuiper Belt fit in with observed dusty disks around other sun-like stars, such as Beta Pictoris?

2. What is the spatial extent of the Kuiper Belt – its radial and inclination distribution? What are the relative populations in the Scattered Disk and the Classical Kuiper Belt? What mechanisms have given rise to the large eccentricities and inclinations in the trans-Neptunian region?

3. What are the relative proportions of the resonant and non-resonant

KBOs, their eccentricity, inclination and libration amplitude distributions? These provide constraints on the orbital migration history of the outer planets.

4. The phase space structure in the vicinity of Neptune's mean motion resonances is reasonably well understood only for the 3:2 resonance. Similar studies of the other resonances are warranted.

5. Given the apparent highly nonuniform orbital distribution of KBOs, precisely what is the source region of short period comets and Centaurs? What is the nature of the long term instabilities that provide dynamical transport routes from the Kuiper Belt to the short period comet and Centaur population?

6. Was the primordial Kuiper Belt much more massive than at present? What, if any, were the mass loss mechanisms (collisional grinding, dynamical stirring by large KBOs or lost planets)?

7. Is there a significant population of Neptune Trojans? Or a belt between Uranus and Neptune? What can we learn from its presence/absence about the dynamical history of Neptune?

8. What is the distribution of spins of KBOs? It may help constrain their collisional evolution.

9. What is the frequency of binaries in the Kuiper Belt? (How unique is the Pluto-Charon binary?)

10. What is the relationship between the Kuiper Belt and the Oort Cloud? How does the mass distribution in the Kuiper Belt relate to the formation of the Oort Cloud? Is there a continuum of small bodies spanning the two regions?

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FIGURE CAPTIONS

Figure 1. Dynamical lifetime before first close encounter with Neptune for test particles with a range of semi-major axes and eccentricities, and with initial inclinations of one degree (based on Duncan et al 1995). Each particle is shown as a narrow vertical strip, centered on the particle's initial eccentricity and semimajor axis. The lightest colored (yellow) strips represent particles that survived the length of the integration (4 billion years). Dark regions are particularly unstable. The dots indicate the orbits of Kuiper belt objects with reasonably well-determined orbits in January, 1999. (Orbits with inclinations less than 10 degrees are shown in red, those with higher inclinations are displayed in green.) The locations of low order mean motion resonances with Neptune and two curves of constant perihelion distance, q are shown.

Figure 2. The locations and widths in the (a, e) plane of Neptune's low order mean motion resonances in the Kuiper Belt. Orbits above the dotted line are Neptune-crossing; the hatched zone on the left indicates the chaotic zone of first order resonance overlap. The dots indicate the orbits of KBOs with reasonably well determined orbits in January 1999. (Adapted from Malhotra (1995).)

Figure 3. The major dynamical features in the vicinity of Neptune's 3:2 mean motion resonance. The locations of the resonance separatrix (dark solid lines), two secular resonances, apsidal ν_8 and nodal ν_{18}, and the Kozai resonance were obtained by a semi-numerical analysis of the averaged perturbation potential of Neptune and the other giant planets. The blue shaded region is the stable resonance libration zone in the unaveraged potential of Neptune. (Adapted from Morbidelli (1997) and Malhotra (1995).)

Figure 4. A map of the dynamical diffusion speed of the semimajor axis of test particles in Neptune's 3:2 mean motion resonance. The test particles in this numerical study had initial inclinations less than 5 degrees. The color scale is indicated on the bottom. (From Morbidelli (1997).)

Figure 5. The distribution of orbital elements of trans-Neptunian test particles after resonance sweeping, as obtained from a 200 Myr numerical simulation in which the Jovian planets' semimajor axes evolve according to $a(t) = a_f - \Delta a \exp(-t/\tau)$, with timescale τ = 4Myr and $\Delta a = \{-0.2, 0.8, 3.0, 7.0\}$ AU for Jupiter, Saturn, Uranus and Neptune, respectively (Malhotra, 1999). The test particles were initially distributed smoothly in circular, zero-inclination orbits between 28 AU and 63 AU, as indicated by the dotted line in the lower panel. In the upper two panels, the filled circles represent particles that remain on stable orbits at the end of the simulation, while the open circles represent the elements of those particles which had a close encounter with a planet and subsequently move on scattered chaotic orbits ('removed' from the simulation at the instant of encounter). Neptune's mean motion resonances are indicated at the top of the figure.

Figure 6. The temporal behavior of a long-lived member of the scattered disk. The black curve shows the behavior of the comet's semi-major axis.

The gray curve shows the perihelion distance. The three dotted curves show the location of the 3:13, 4:7, and 3:5 mean motion resonances with Neptune.

Figure 7. The surface density of comets beyond Neptune for two different models of the source of Jupiter-family comets. The dotted curve is a model assuming that the Kuiper belt is the current source (Levison & Duncan 1997). There are 7×10^9 comets in this distribution between 30 and 50 AU. This curve ends at 50 AU because the models are unconstrained beyond this point and not because it is believed that there are no comets there. The solid curve is DL97's model assuming the scattered disk is the sole source of the Jupiter-family comets. There are 6×10^8 comets currently in this distribution.

Figure 8. A strawman model of the Kuiper belt. The dark area at left marked "Planets" shows the distribution of solid material in the outer planets region: it follows a power law with a slope of about -2 in heliocentric distance. The dashed curve is an extension into the Kuiper belt of the power law found for the outer planets and illustrates the likely initial surface density distribution of solid material in the Kuiper belt. The dashed curve has been scaled so that it is twice the surface density of the outer planets, under the assumption that planet formation was 50% efficient. The solid curve shows a model of the mass distribution by Duncan et al. (1995, see Figure 7), scaled to an estimated population of 5×10^9 Kuiper belt objects between 30 and 50 AU. The Kuiper belt is divided into three regions:, see text for a description. The dotted curves illustrate the unknown shape of the surface density distribution in Region B.

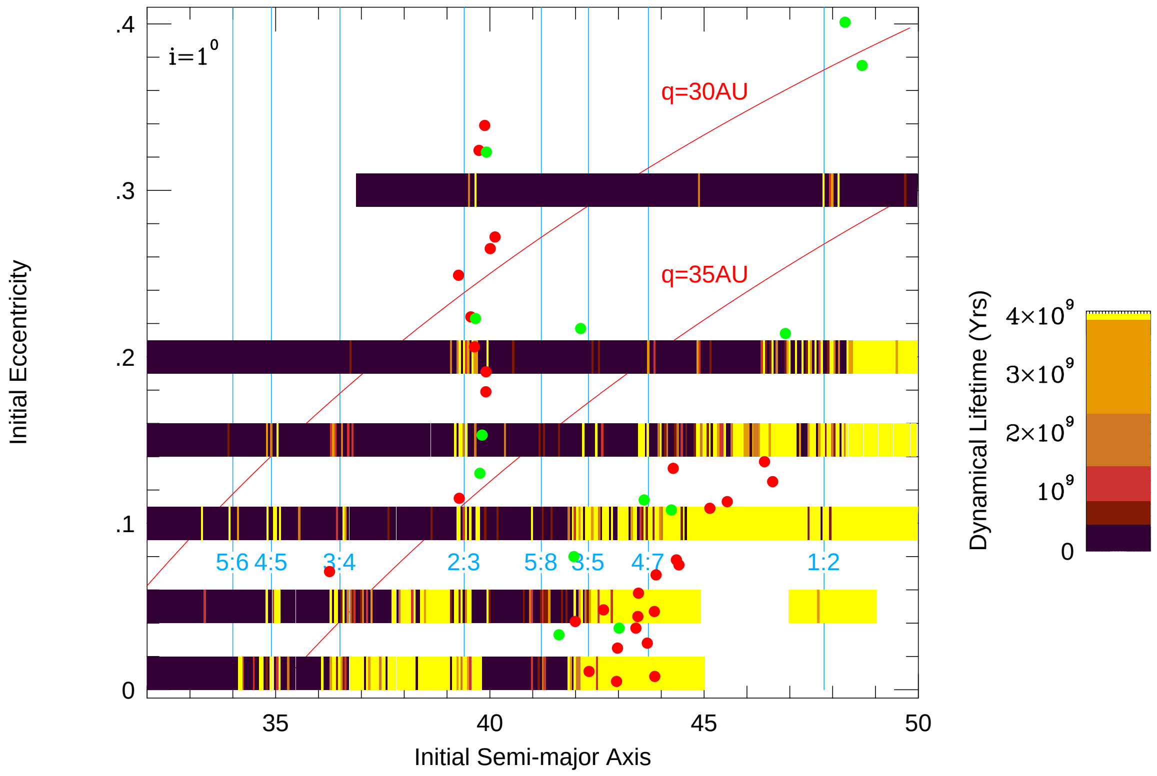

i=1⁰
q=30AU
q=35AU
5:6 4:5 3:4 2:3 5:8 3:5 4:7 1:2
Initial Eccentricity
Initial Semi-major Axis
Dynamical Lifetime (Yrs)
4×10⁹
3×10⁹
2×10⁹
10⁹
0

Figure 1

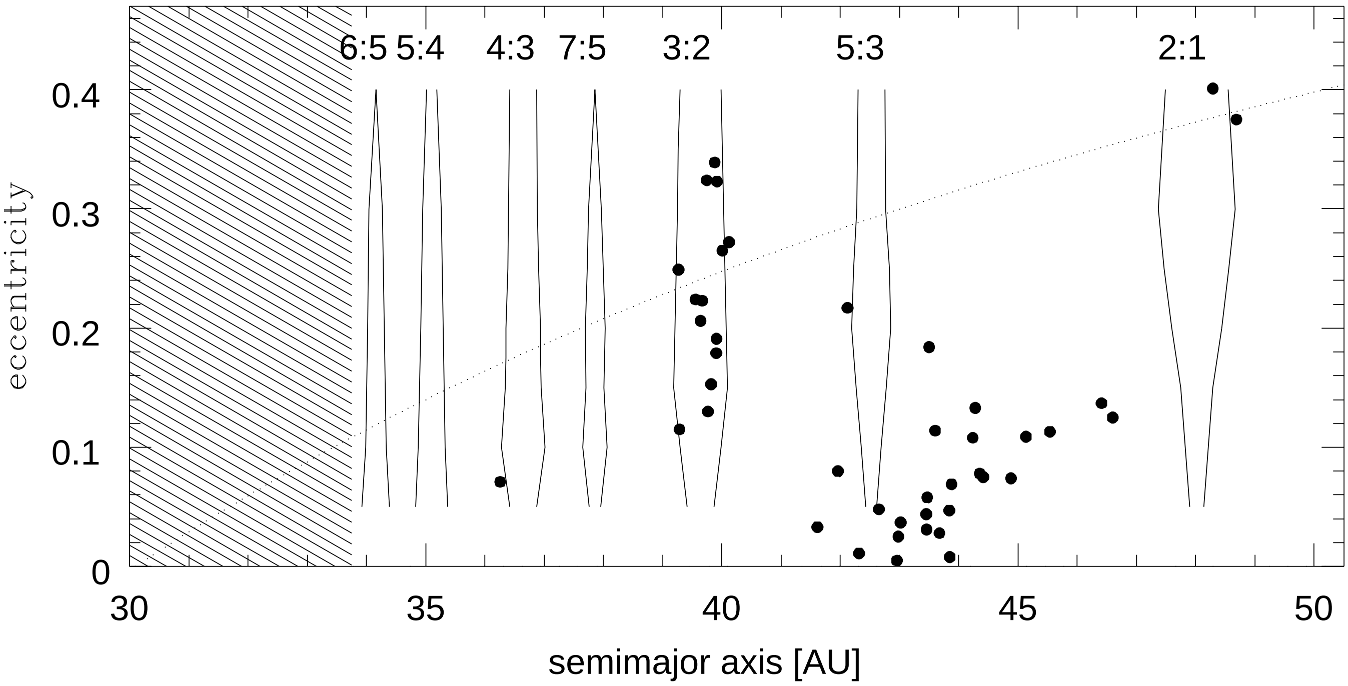

6:5
5:4
4:3
7:5
3:2
5:3
2:1
0.4
0.3
0.2
0.1
0
30
35
40
45
50
eccentricity
semimajor axis [AU]

Figure 2

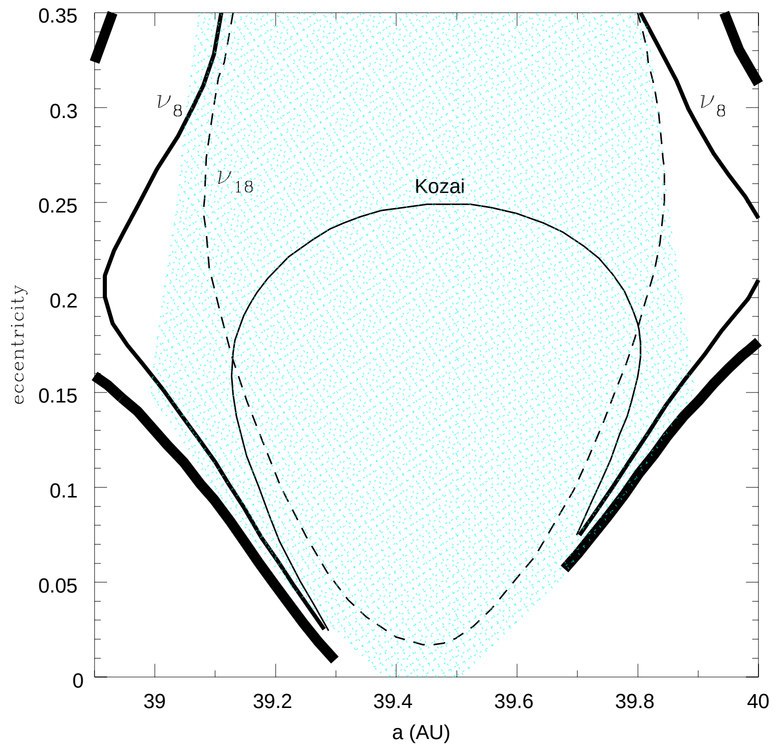


Figure 3

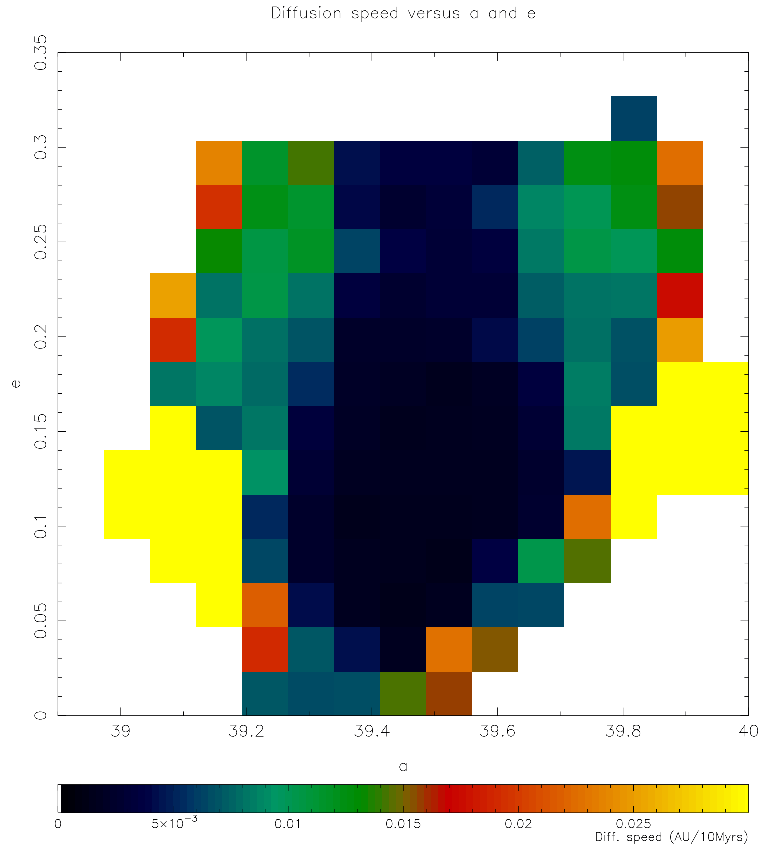


Figure 4

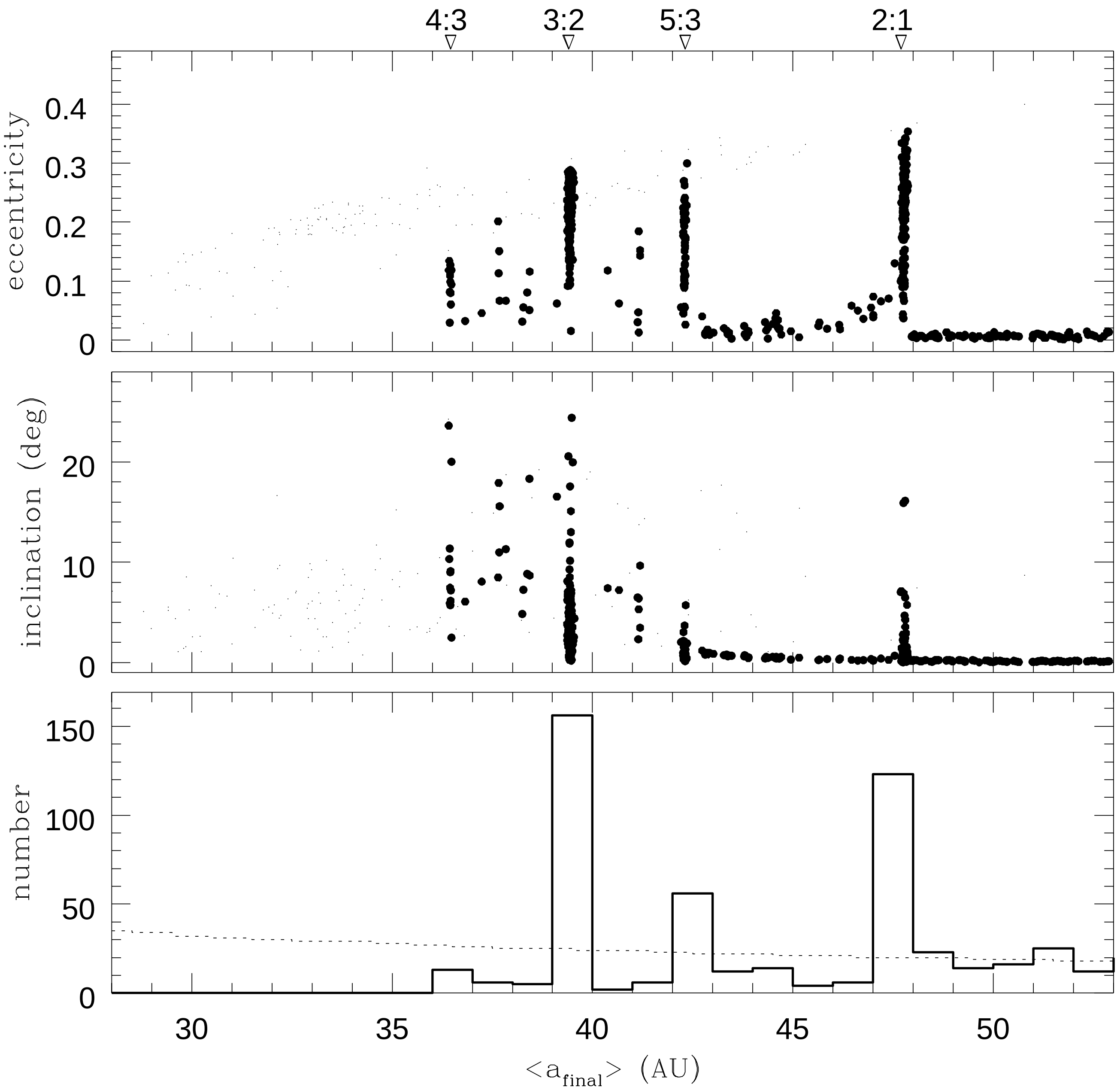

4:3
3:2
5:3
2:1
eccentricity
0.4
0.3
0.2
0.1
0
inclination (deg)
20
10
0
number
150
100
50
0
30
35
40
45
50
<a_final> (AU)

Figure 5

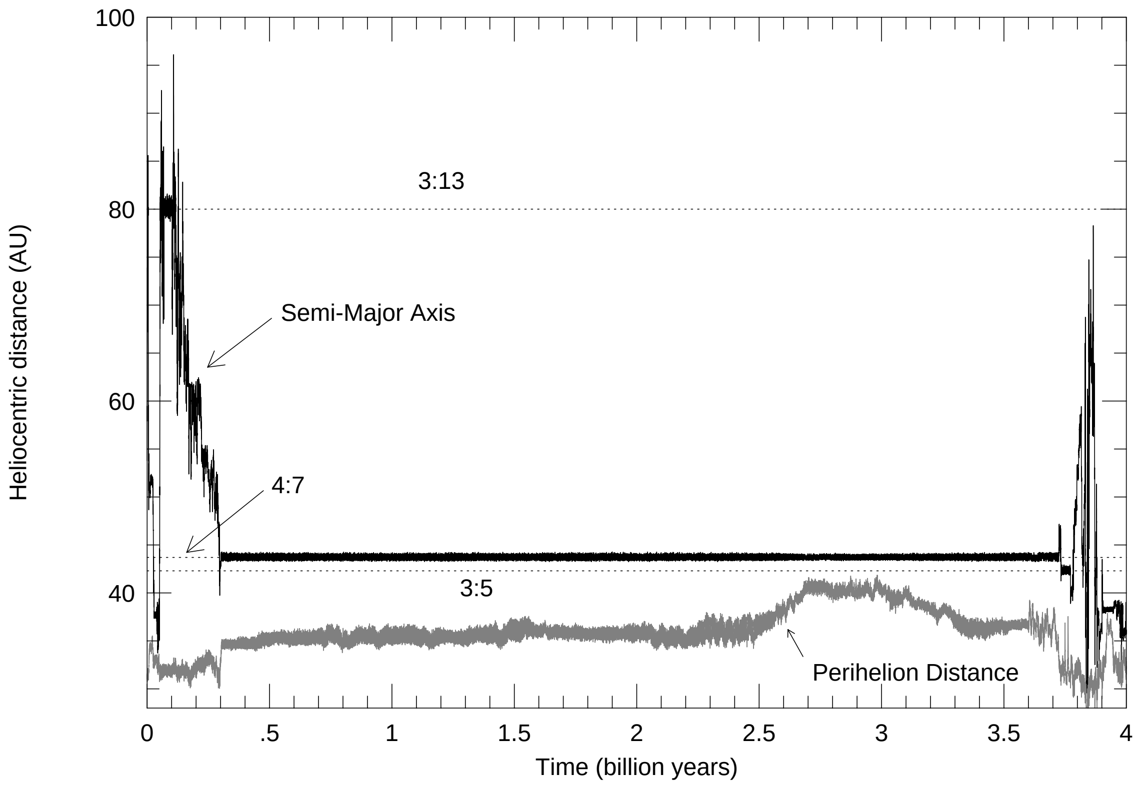
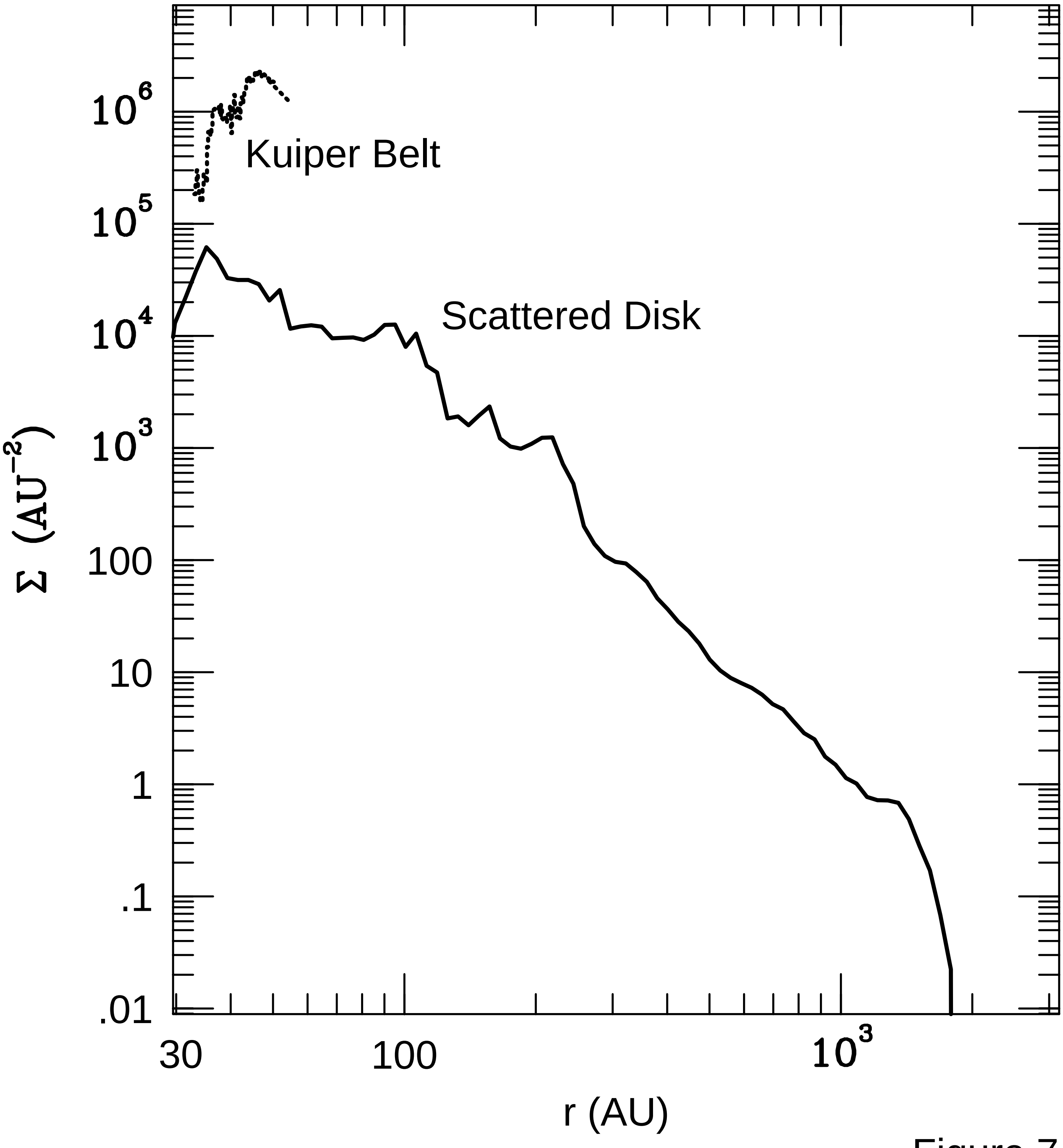

100
80
60
40
Heliocentric distance (AU)
3:13
Semi-Major Axis
4:7
3:5
Perihelion Distance
0
.5
1
1.5
2
2.5
3
3.5
4
Time (billion years)

Figure 6



10^6
10^5
10^4
10^3
100
10
1
.1
.01
Σ (AU^{-2})
Kuiper Belt
Scattered Disk
30
100
10^3
r (AU)

Figure 7

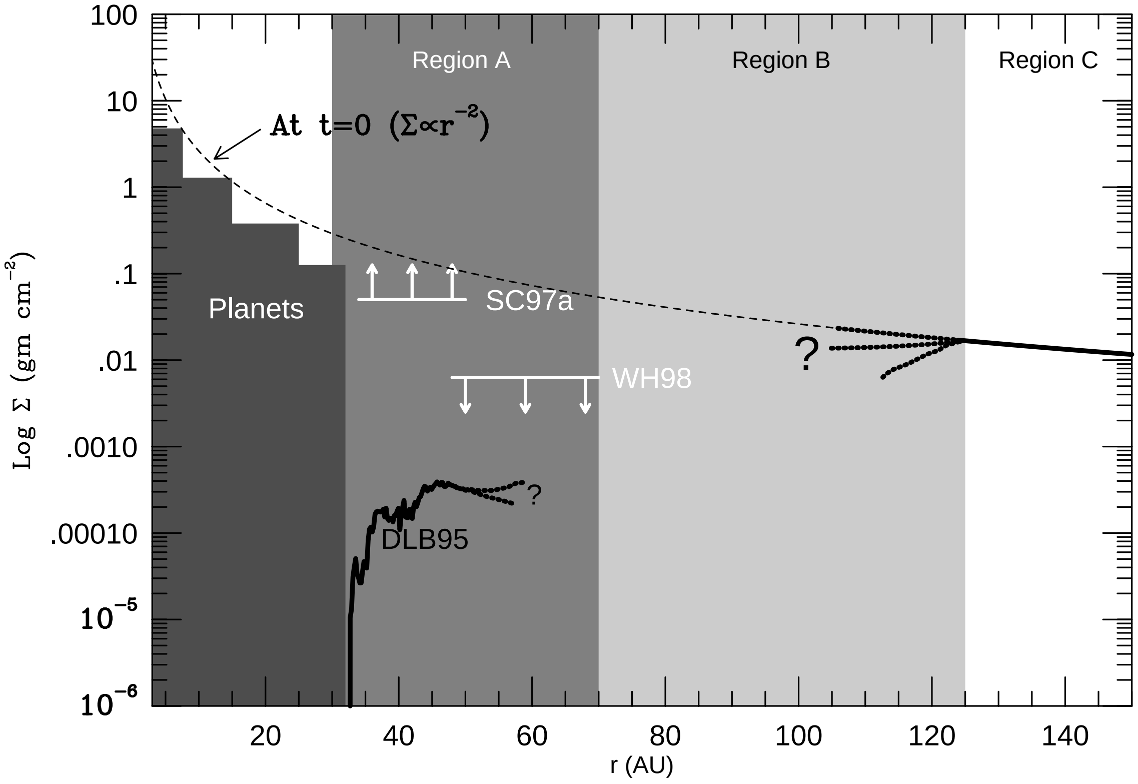

100
10
1
.1
.01
.0010
.00010
10^{-5}
10^{-6}
Log Σ (gm cm^{-2})
20
40
60
80
100
120
140
r (AU)
Region A
Region B
Region C
At t=0 (Σ∝r^{-2})
Planets
SC97a
WH98
DLB95
?
?

Figure 8