Method 1: Fourier transform of refractivity profiles

Spikes seen in stellar occultations arise from small scale refractivity variations, which can be quantified by performing Fourier transforms on refractivity profiles derived from lightcurve inversion. This method has the advantage that the technique of lightcurve inversion, as well as that of Fourier analysis, is well established. Fourier analysis of temperature or refractivity profiles have been performed for Titan, Saturn, and Uranus. Here we review this method, with a constructed example.

Sicardy et al. 1999. The structure of Titan's stratosphere from the 28 Sgr occultation. *Icarus* **142**, 357-390.

Foust 1999. Occultation studies of Saturn's upper atmosphere. Ph. D. thesis, MIT.

Young, 2001. Observational evidence for breaking gravity waves in stratospheres of the outer solar system. Presented at CEDAR 2001 meeting, Boulder CO, June 2001

Relating refractivity in a planet's atmosphere with flux observed at Earth

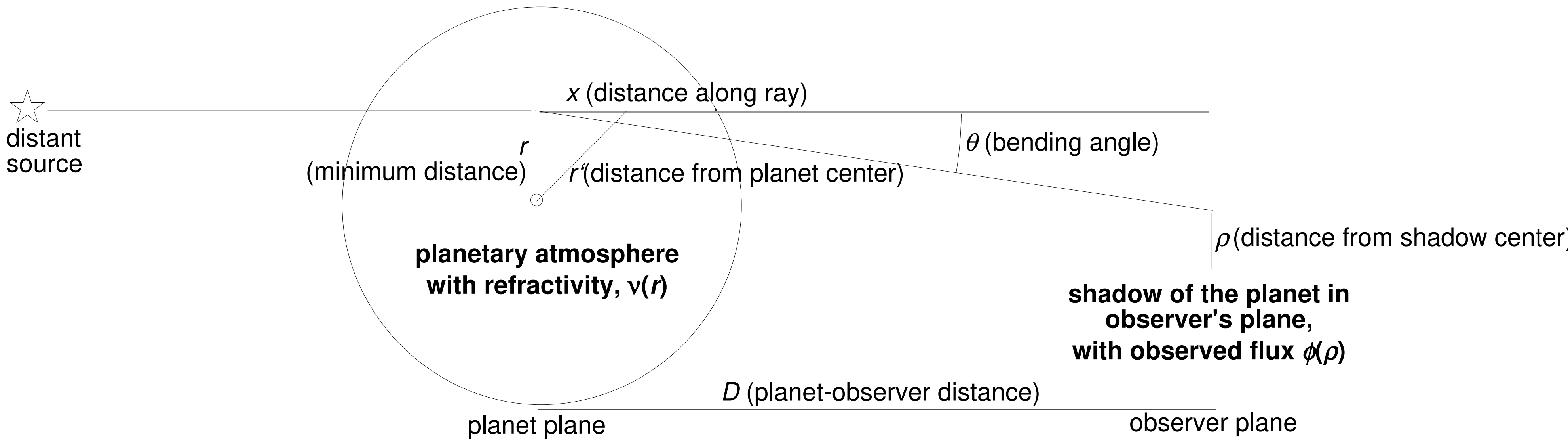

Fig. 3. Geometry of an occultation.

During a stellar occultation, differential refraction, $d\nu(r)/dr$, in a planet's atmosphere defocuses incident starlight, leading to decreased flux in the planet's shadow, $\phi(\rho)$, observed at Earth. Small fluctuations in $\nu(r)$ causes local refocusing, leading to sharp spikes in the observed flux.

Under some standard assumptions, the flux, $\phi(\rho)$, can be calculated from the refractivity, $\nu(r)$, using **forward modeling**:

$$\theta(r) = \frac{d}{dr}\int_{-\infty}^{+\infty} \nu(r')dx \qquad \rho(r) = r + D\theta(r) \qquad \phi(\rho) = \left|\frac{dr}{d\rho}\right|$$

Assuming $dr/d\rho > 0$ (no ray crossing), refractivity can be calculated from flux using **lightcurve inversion**:

$$\theta(\rho) = -\frac{1}{D}\int_{\rho}^{\infty} 1-\phi(\rho')d\rho' \qquad r(\rho) = \rho - D\theta(\rho) \qquad \nu(r) = -\frac{1}{2\pi r}\int_{-\infty}^{+\infty}\theta(r')dx$$

Wasserman and Veverka 1973, On the reduction of occultation light curves. *Icarus* **20**, 322-345.

For simplicity, these equations ignore oblateness, relativistic bending, small planet terms, diffraction, finite star size, or horizontal inhomogeneity.

An example atmosphere and lightcurve

Given an input function for the power spectrum of normalized refractivity perturbations, we have generated a random refractivity profile and a lightcurve. Our goal is to show how well three different methods reproduce both the refractivity profile and its underlying power spectrum.

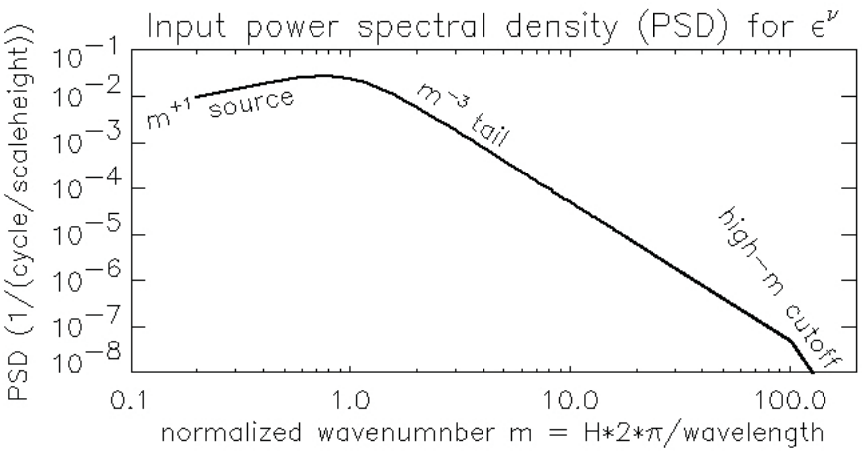

Fig. 4. Input power spectrum for normalized refractivity, based on the Desaubies function for saturated gravity waves.

Smith, S. *et al.* 1987. Evidence for a saturated spectrum of atmospheric gravity waves. *J. Atmos. Sci* **44**, 1404-1410.

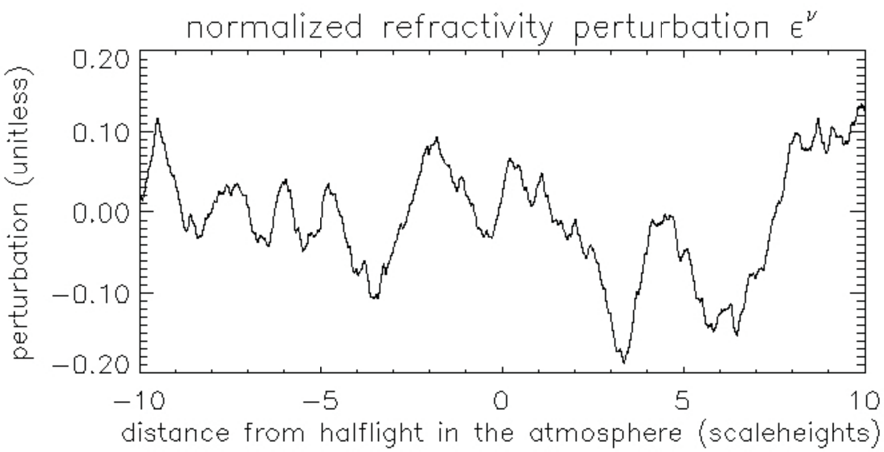

Fig. 5. For random phases, a sample refractivity perturbation at high resolution (with a spacing of scaleheight/512).

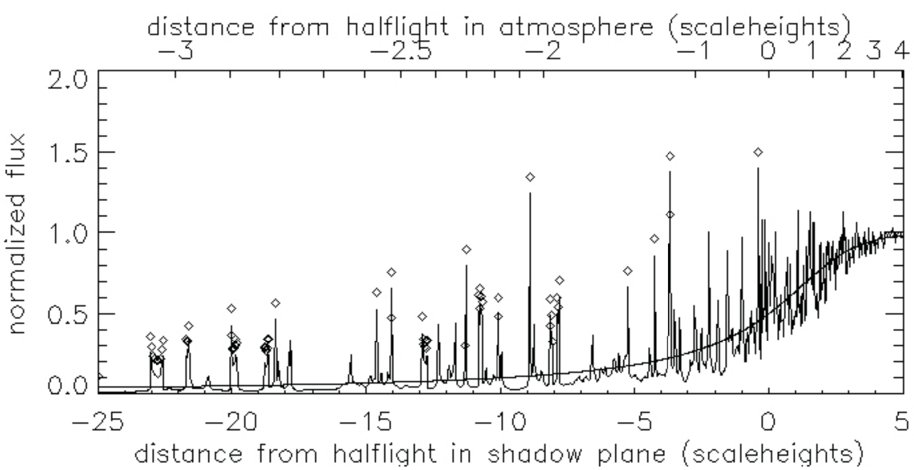

Fig. 6. The lightcurve generated by the refractivity in Fig. 5, assuming geometric optics, with a spacing of H/32 at the upper baseline. Diamonds mark locations of ray crossing.

Inverting the example lightcurve, and calculating the Fourier transform

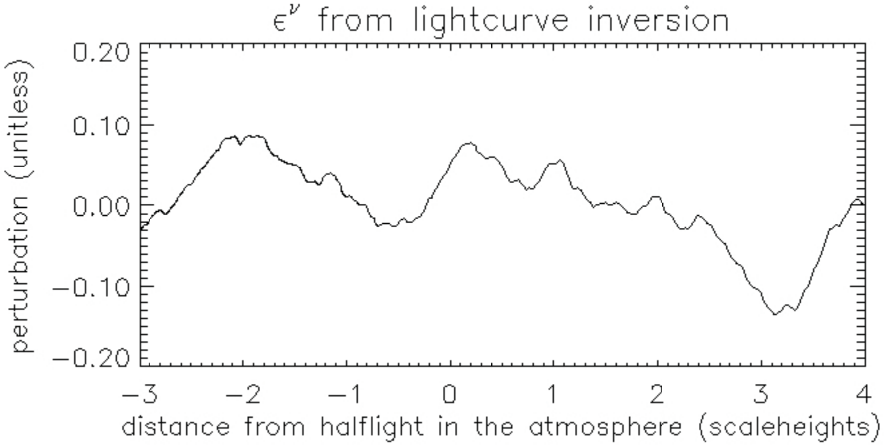

Fig. 7. Refractivity variation (ε^ν) from lightcurve inversion.

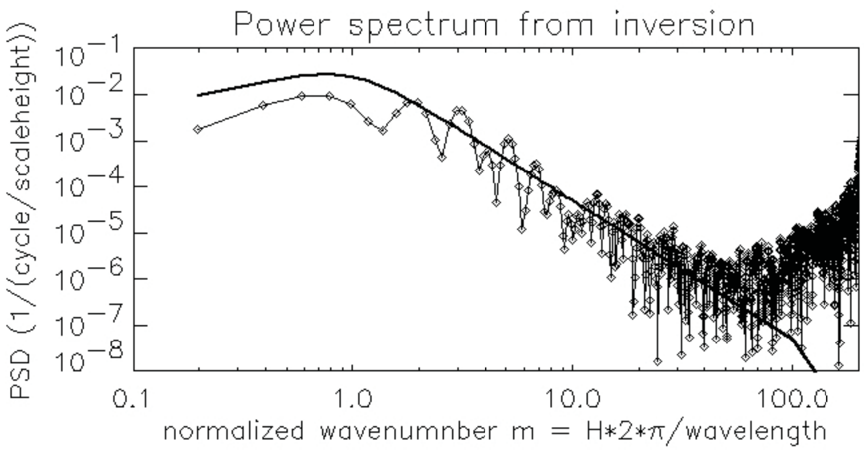

Fig. 8. Fourier transform of ε^ν from inverted lightcurve.

Using the equations for lightcurve inversion, the refractivity in the atmosphere can be derived from the observed occulted stellar flux, over the range of a few scale heights. The resulting refractivity is not evenly sampled—while the first 100 radii have an average resolution >600 points per scaleheight, the last 100 radii have a resolution of only 35 points per scaleheight (for a Nyquist wavenumber of 110), This leads to aliasing at higher wavenumbers.