Method 2: Wavelet transform of refractivity profile

The wavelet transform is an alternative to the Fourier transform for studying the power spectra of gravity waves. It is especially appealing for stellar occultations, where the vertical resolution changes drastically over the span of the inverted refractivity profile. By using basis sets that are spatially compact, we can choose to include, for each altitude range, only those wavelets that are broad enough to be resolved. We repeat the analysis of method 1, using narrower wavelets below halflight than above.

Deriving wavelet coefficients

By choosing orthonormal wavelets (as in Fig, 2), we can decompose a function into sums over wavelets. The form of the wavelet transform is similar to that of the Fourier transform, which can be thought of as a basis set of sines and cosines.

Fourier Transform over $[0, 2\pi]$

$$f(x) = \sum_{m=-\infty}^{\infty} F_m e^{imx}$$

$$F_m = \int_0^{2\pi} f(x) e^{-imx} dx$$

Wavelet Transform

$$f(x) = \sum_{j,k=-\infty}^{\infty} F_{j,k} \left\{ 2^{j/2} \psi\left(2^j x - k\right) \right\}$$

$$F_{j,k} = \int_{-\infty}^{\infty} f(x) \left\{ 2^{j/2} \psi^*\left(2^j x - k\right) \right\} dx$$

In this example, the mother wavelet (j=0, k=0) is four scaleheights wide, and spans -3H to +1H. The next finer wavelet, j=1, k=0 and k=1, is half as wide, has twice the characteristic frequency, and requires twice as many wavelets to span the same range (-3H to -1H, -1H to +1H). This continues for j=2, k=0,1,2,3 etc., with the frequency doubling at each stage.

The wavelet spectrum of refractivity.

The wavelet spectrum consists of the square of the coefficients, $F_{j,k}$. To compare the wavelet spectrum with the Fourier spectrum (Fig.s 4 and 8), average the coefficients for a particular scale (e.g., j) over position (e.g. k). If averaged over enough spatial positions, the wavelet spectrum equals a smoothed version of the Fourier spectrum, integrated over the square of the wavelet in frequency space.

Perrier *et al.* 1995. Wavelet spectra compared to Fourier Spectra. *J. Math. Phys.* **36**. 1506-1519.

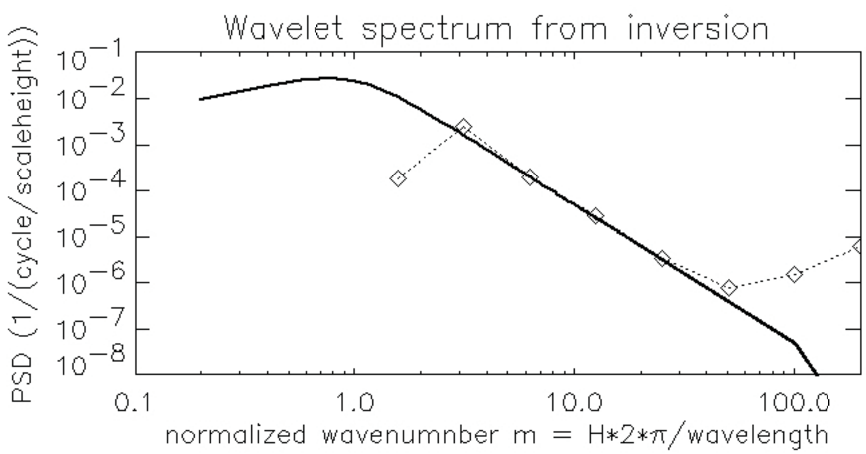

Fig 9. The wavelet spectrum of the refractivity derived from lightcurve inversion

The wavelet spectrum of the inverted refractivity reproduces the input spectrum for j=1 to 5. At j=0, the wavelet spectrum is dominated by edge effects, while for j > 5, the derived spectrum is still too high (perhaps from aliasing).