Method 3: Wavelet transform from the lightcurve itself

Lightcurve inversion has several drawbacks. First, it assumes no ray crossing, a negligible stellar diameter, spherical symmetry, and geometric optics. Second, neighboring points in the derived temperatures or refractivities are highly correlated. Finally, refractivities are retrieved over a limited vertical range, exacerbating edge effects. A wavelet decomposition of the refractivity, combined with forward modeling (so a model lightcurve is compared directly to observed fluxes) can attack these limitations. The game comes down to finding a set of wavelet coefficients for the refractivity perturbation that reproduces the observed lightcurve.

Calculating lightcurves given the wavelet coefficients.

To find coefficients by forward modeling, we need to calculate lightcurves from wavelength coefficients hundreds of times. Calculating a refractivity from a set of coefficients, and then integrating the refractivity along the line of sight to find the bending angle, θ, would be prohibitively slow. Fortunately, the bending angle and its derivative are linear functions of the refractivity. This means we can directly calculate θ from the refractivity's wavelet coefficients.

To show how this works, first write the expression for the refractivity explicitly in terms of its wavelet coefficients and the wavelets from Fig. 2, which we designate ψ^{ν}.

$$\begin{aligned} \nu(r) &= \overline{\nu}(r)\left[1+\varepsilon^{\nu}(r)\right] \\ &= \overline{\nu}(r) + \sum_{j,k} \overline{\nu}(r)\left[F_{j,k}\psi^{\nu}_{j.k}(r)\right] \end{aligned}$$

$$\overline{\nu}(r) = \nu_h e^{-(r-r_h)/H}$$

Taking this sum term by term, we can write the refractivity explicitly in terms of the *same* wavelet coefficients and a new wavelet, which we designate ψ^{θ}.

$$\theta(r) = \bar{\theta}(r) + \sum_{j,k} \bar{\theta}(r)\left[F_{j,k}\psi^{\theta}_{j.k}(r)\right] \qquad \bar{\theta}(r)\psi^{\theta}_{j,k}(r) = \frac{d}{dr}\int_{-\infty}^{+\infty} \bar{v}(r')\psi^{v}_{j,k}(r')dx$$

The Fourier transform of the bending angle wavelet is easily expressed in in terms of the refractivity wavelet:

$$\hat{\psi}^{\theta}_{jk}(m) = \frac{1 - iHm}{\left(H^2m^2 + 1\right)^{1/4}}\sqrt{\frac{2 + iHm}{2 - iHm}}\,\hat{\psi}^{v}_{jk}(m)$$

Equivalent expressions can be written to quickly calculate phase delay and the derivative of the bending angle.

Estimating the refractivity wavelet spectrum directly from the lightcurve.

Because the bending angle of the refractivity appears in the denominator of the equation for flux, the flux is a nonlinear function of the wavelet coefficients. This means that the first step in finding wavelet coefficients is to find good initial conditions. A straight-forward way to do this is to minimize the sum of squared residuals for each wavelet coefficient independently. This will work well if the wavelets are not strongly correlated. From these independent estimates, we can calculate the wavelet spectrum as before.

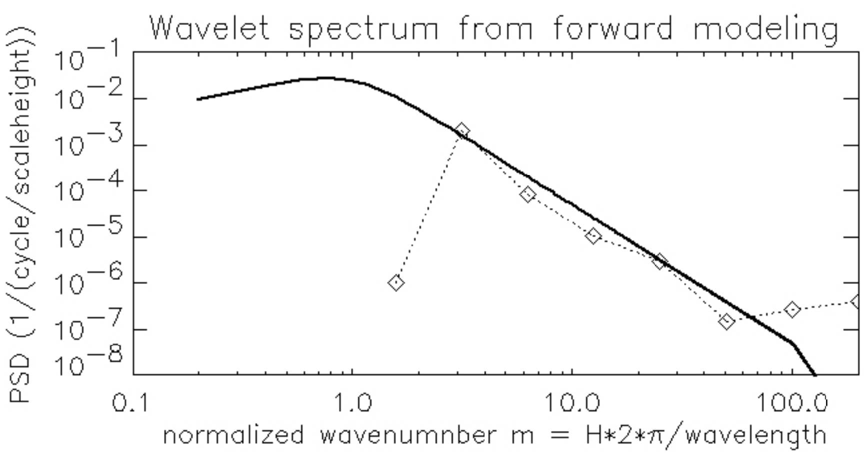

Fig 10. The wavelet spectrum of the refractivity derived from forward modeling.

The wavelet spectrum of the lightcurve again reproduces the input spectrum for j=1 to 5. The high-m turnup is slightly better, but the low-m underestimate is worse.

The example presented here is greatly simplified, but the forward modeling technique can, in practice, be used with finite stellar sizes, wave optics, or horizontal inhomogeneity.

The refractivity profile is not unique, because of ray crossing. The coefficients for wavelets involved in the ambiguity are identifiable by their correlation coefficients, and a choice can be made based on quanitifyible criteria, such as minimizing the total refractivity (or temperature) variance.