Tidal Interaction between a Planet and a Circumplanetary Disk. Wm. R. Ward and R. M. Canup, Dept. of Space Studies, Southwest Research Institute, Boulder, CO 80302

Satellite formation as a byproduct of large scale planetary impacts is the likely origin of the Earth's moon [1-4] and is a strong contender for the formation of the Pluto-Charon pair as well [5,6]. Some binaries in the asteroid and Kuiper belts might also be the result of impacts. If the process passes through an intermediate stage involving a Roche interior disk, the majority of this material is driven into the primary as the disk viscously spreads. Here we describe a tidal mechanism that could inhibit inward diffusion and increase the fraction of material available for satellite formation.

The tidal interaction between a planet and an isolated satellite is a well-studied process that leads to the transfer of angular momentum between the planet's spin and the orbit of the satellite. When modeling the tidal evolution of systems of multiple satellites, the standard assumption is that, on average, each satellite interacts only with its own tidal bulge and not with tidal bulges raised by other satellites. However, this is not necessarily a good assumption when the motions of satellites are correlated (e.g., in a mean-motion resonance) or when interactions among all orbiting bodies lead to collective behavior. In such cases, the cross-interactions among all orbiting bodies and their tides need to be considered. For example, non-axisymmetric disk structures such as spiral waves could themselves raise tides and affect angular momentum exchange. The ultimate efficacy of the mechanism will depend on the complex behavior of a specific disk. Here we are content to determine the tidal torque due to a generic spiral wave of the form,

$$\sigma_1(r,\theta) = \sigma_1(r)\cos[\int k(r)dr + m(\theta - \Omega_{ps}t)], \quad (1)$$

which illustrates some of the interesting features of the process. In this expression, σ_1 represents the surface density perturbation, $k(r)$ is the wavenumber, m is the number of spiral arms, and Ω_{ps} is the pattern speed of the wave.

We begin by writing down the standard gravitational potential at a point on the planet's equator (R, θ) due to an orbiting mass,

$$dM = \sigma_1(r',\theta')r'd\theta'dr' \quad (2)$$

at (r', θ') [7], *i.e.*,

$$d\Phi(R,\theta) = \frac{GdM'}{r'}\sum_{n=2}^{\infty}\left(\frac{R}{r'}\right)^n P_n(\cos(\theta' - \theta)) \quad (3)$$

A useful expansion of the Legendre polynomial $P_n(\cos\phi)$ is [8]

$$\sum_{l=0}^{L}\frac{2\Gamma(l+1/2)\Gamma(n-l+1/2)}{\pi\Gamma(l+1)\Gamma(n-l+1)}\cos[(n-2l)\phi] \quad (4)$$

with $\Gamma(z)$ representing the Gamma function, $L = (n/2)$ for n even, and $(n-1)/2$ for n odd. The total potential, $\Phi(R,\theta) = \int_{disk} d\Phi$, is found by integrating over the disk. Interestingly, it turns out that only $n \geq m$ contribute and even (odd) m will interact only with even (odd) Legendre polynomials. For a given m we get $\Phi_m \equiv \sum_{l=0}^{\infty}\Phi_{m,l}$, where

$$\Phi_{m,l}(R,\theta) = \pi G\sigma_o R\sum_{l=0}^{\infty}a_{m,l} \quad (5)$$

$$\times \int_1^{r_R/R}\hat{\sigma}_1\left(\frac{R}{r'}\right)^{m+2l}\cos\left[\int k dr' + m\theta\right]d\left(\frac{r'}{R}\right)$$

with $\hat{\sigma}_1 \equiv \sigma_1(r')/\sigma_o$ and

$$a_{m,l} \equiv \frac{2}{\pi}\frac{\Gamma(l+1/2)\Gamma(m+l+1/2)}{\Gamma(l+1)\Gamma(m+l+1)} \quad (6)$$

The primary's tidal response potential for each component, $\Phi_{m,l}$, is of the form,

$$U_{m,l}(r,\theta) = k_L\Phi_{m,l}(R,\theta-\delta)\left(\frac{R}{r}\right)^{m+2l+1} \quad (7)$$

where k_L and δ are the Love number and tidal lag angle respectively [7]. The final step is to integrate the torque exerted on a mass element of the disk,

$$dT_m = dM\Sigma_{l=0}^{\infty}(\partial U_{m,l}/\partial\theta) \quad (8)$$

over the disk.

There is an important subtlety, however, involving the lag angle. The usual procedure is to assume δ is proportional to the forcing frequency as seen from the surface of the planet rotating at frequency ω. For an individual moonlet with mean motion Ω, $\delta \propto \omega - \Omega$, indicating the well known result that moonlets exterior (interior) to corotation have a leading (trailing) tidal bulge and gain (lose) angular momentum. On the other hand, if the nonaxisymmetric signature of the disk rotates at a constant pattern speed, Ω_{ps}, the phase lag $\delta \propto \omega - \Omega_{ps}$ everywhere, irrespective of the local orbital mean motion. Particles stream in and out of the spiral arms, but are continuously replaced so that only the pattern's rotation produces a discernable time variation in the total potential. For a constant δ, the total torque is

$$T_m = mk_L\sin(m\delta)[G(\pi\sigma_o R^2)^2/R]\sum_{l=0}^{\infty}a_{m,l}A_{m,l}^2 \quad (9)$$

where

$$A_{m,l} \equiv \int_1^{x_R}\hat{\sigma}_1(x)x^{-(m+2l)}e^{i\int\hat{k}dx}dx \quad (10)$$

with $\hat{k} \equiv kR$. By comparison, the angular momentum flux due to the gravitational torque from an m-armed wave is of order [9] $\mathcal{F} \approx m\pi^2G\sigma_1^2R^3/\hat{k}^2$.

Both the Love number and sin($m\delta$) could be large for the post-impact planet in which a debris disk is generated. This suggests that if the disk evolves primarily via gravitational torques from low m waves, their tidal interaction with the planet could modify the disk angular momentum during the process. Since a disk tends to fragment beyond the Roche distance, both its outer edge and the planet's surface are boundaries that could reflect waves and lead to a standing wave pattern. This can be regarded as a superposition of outward traveling trailing waves and inward traveling leading waves. However, the torque of eqn. (9) does not depend on the sign of k so that the total torque is found by doubling T_m.

Is there a mechanism that could excite large amplitude density waves in a circumplanetary disk not subject to strong satellite perturbations? One possibility is the so-called viscous over-stability [10,11]. This causes the amplitude of a traveling wave to grow if $d(\sigma\nu)/d\sigma > 0$, where ν is the disk viscosity. If the effective viscosity is induced by incipient gravitational instability [12], $\nu_{eff} \approx G^2\sigma^2/\Omega^3$, and there is a strong positive gradient of order $\sim 3\nu_{eff}$, which should lead to growth. Thus even weak perturbations by a satellite starting to form beyond the Roche distance could be amplified. If the growth is limited by non-linear effects, the amplitudes could be substantial and the planet could tidally deposit angular momentum into the disk, resulting is a smaller fraction diffusing inward.

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